TUHH
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Institute of Embedded Systems

Compilers for Embedded Systems

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Chapter 4

Pre-Pass Optimizations

Outline

1. Introduction & Motivation
2. Compilers for Embedded Systems – Requirements & Dependencies
3. Internal Structure of Compilers
4. **Pre-Pass Optimizations**
5. HIR Optimizations and Transformations
6. Code Generation
7. LIR Optimizations and Transformations
8. Register Allocation
9. WCET-Aware Compilation
10. Outlook

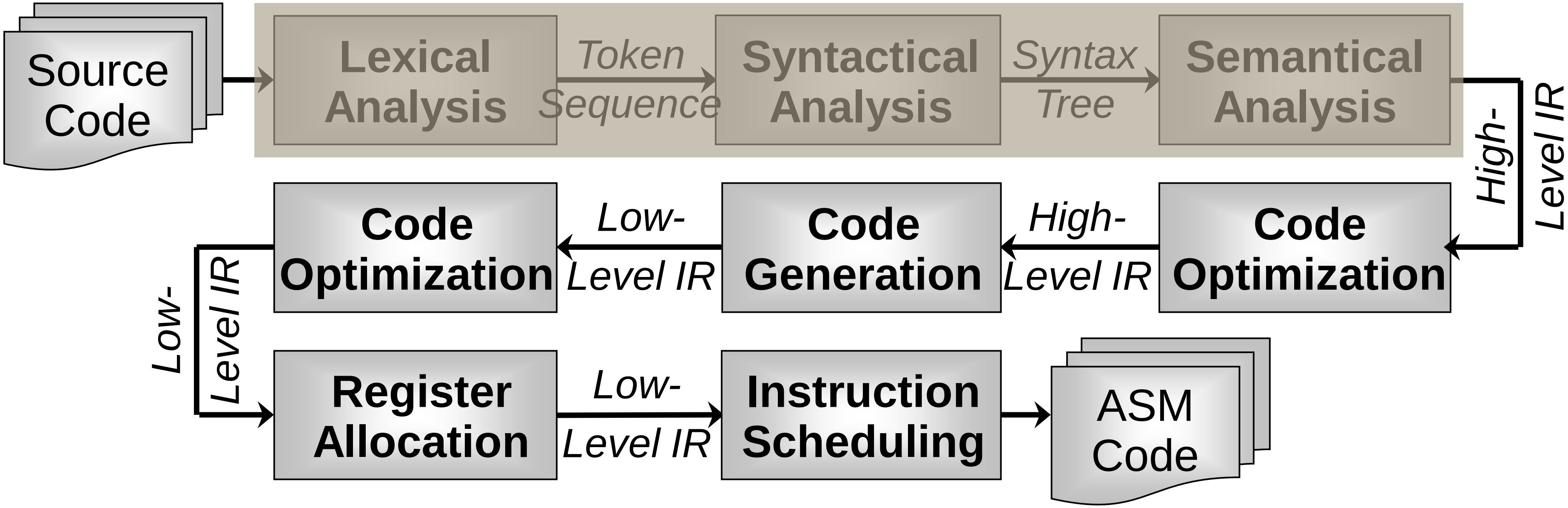
Chapter Contents

4. Pre-Pass Optimizations

- Motivation of Pre-Pass Optimizations
- Loop Nest Splitting
    - Embedded Multimedia: MPEG 4 Motion Estimation
    - Workflow of Loop Nest Splitting
    - Condition Satisfiability
    - Condition Optimization & Genetic Algorithms
    - Search Space Generation
    - Search Space Exploration
    - Results (ACET, Energy, Code Size)

Motivation of Pre-Pass Optimizations (1)

☞ ***Retrospect: Structure of an Optimizing Compiler with 2 IRs:***



Source Code

Lexical Analysis

Token Sequence

Syntactical Analysis

Syntax Tree

Semantical Analysis

High-Level IR

Code Optimization

High-Level IR

Code Generation

Low-Level IR

Code Optimization

Low-Level IR

Register Allocation

Low-Level IR

Instruction Scheduling

ASM Code

Question: May only the compiler optimize code?

Motivation of Pre-Pass Optimizations (2)

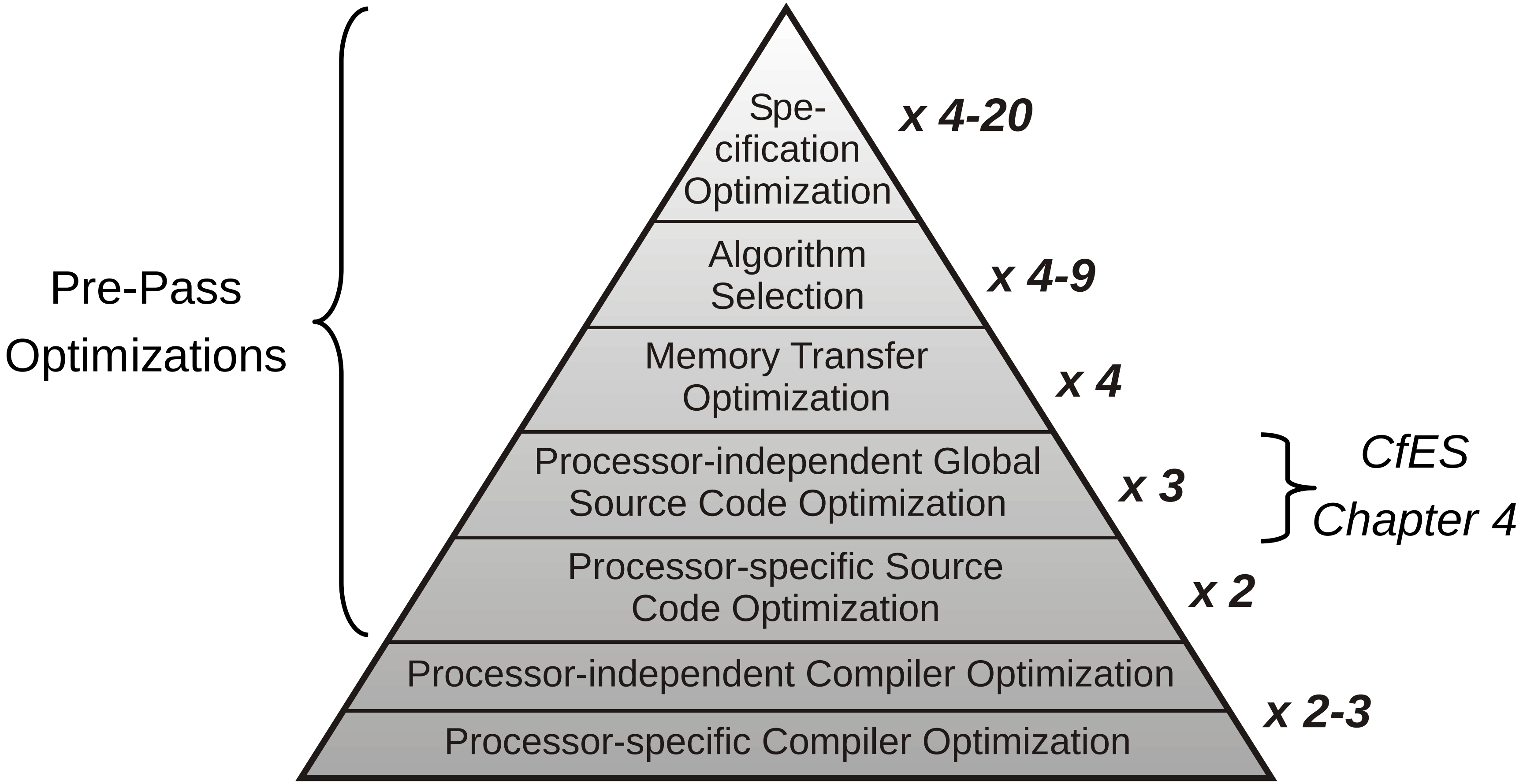
Optimizations outside a compiler are called

- *Post-pass* if they take place *after* the compiler
- *Pre-pass* if they take place *before* the compiler

Advantages of Pre-Pass Optimizations

- Source code transformations more easily comprehensible.
- Allow for manually “playing” with an optimization technique before a laborious implementation.
- Independent of the actual compiler due to source code-level; principally applicable for each individual compiler of the source language.
- Due to source code-level independent of the actual target architecture; principally applicable for arbitrary architectures.

Abstraction Levels of Optimizations


Pre-Pass Optimizations
Spe-cification Optimization
x 4-20
Algorithm Selection
x 4-9
Memory Transfer Optimization
x 4
Processor-independent Global Source Code Optimization
x 3
CfES Chapter 4
Processor-specific Source Code Optimization
x 2
Processor-independent Compiler Optimization
x 2-3
Processor-specific Compiler Optimization

Chapter Contents

4. **Pre-Pass Optimizations**
   - Motivation of Pre-Pass Optimizations
   - Loop Nest Splitting
     - Embedded Multimedia: MPEG 4 Motion Estimation
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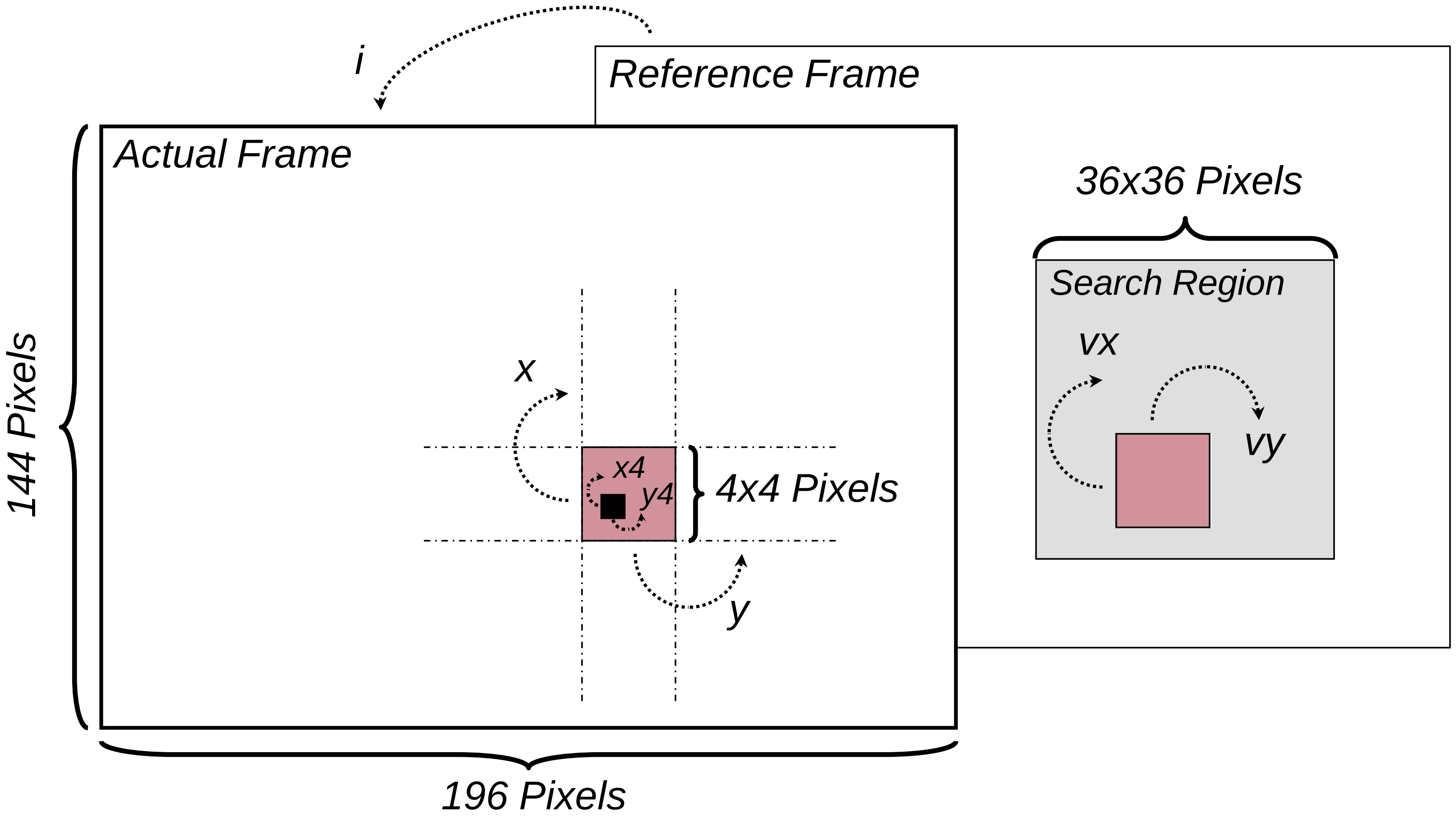
Application Domain of Loop Nest Splitting

Embedded Multimedia Applications

- *Data flow dominated*, i.e., get large amounts of data as inputs, compute large volumes of data as output
  (in contrast to control flow dominated control applications).
- Largest part of the run-time consumed by (deeply) *nested loops*
- *Simple loop structures* with statically known or analyzable lower and upper bounds
- Manipulation of large, multi-dimensional arrays
- Typical example: Streaming applications like MPEG4

Example: MPEG4 Motion Estimation



Source Code MPEG4 Motion Estimation

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    for (y = 0; y < 49; y++)
      for (vx = 0; vx < 9; vx++)
        for (vy = 0; vy < 9; vy++)
          for (x4 = 0; x4 < 4; x4++)
            for (y4 = 0; y4 < 4; y4++) {
              if (4*x+x4<0 || 4*x+x4>35 ||
                  4*y+y4<0 || 4*y+y4>48)
                then_block_1; else else_block_1;
              if (4*x+vx+x4-4<0 || 4*x+vx+x4-4>35 ||
                  4*y+vy+y4-4<0 || 4*y+vy+y4-4>48)
                then_block_2; else else_block_2; }
```

Observations

When Compiling and Executing this Source Code

– Execution of 91,445,760 if-statements in total

– Very irregular control flow due to if-statements

– Additional arithmetical overhead:
Multiplications, additions, comparisons, logical or, ...

☞ *Performance of this code limited by control flow, and not by the computation of motion vectors!*

Loop Nest Splitting

Automated Loop & If-Statement Analysis

– **`x`**, **`y`**, **`x4`** and **`y4`** never carry values such that conditions **`4*x+x4<0`** and **`4*y+y4<0`** are ever true.

☞ *Conditions can be replaced by constant truth value ‘0’.*

– For **`x`** ≥ 10 or **`y`** ≥ 14, both if-statements are always satisfied so that their then-parts are always executed.

☞ *For more than 92% of all executions of the innermost* **`y4`***-loop, both if-statements are satisfied.*

Source Code after Loop Nest Splitting

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    for (y = 0; y < 49; y++)
      if (x >= 10 || y >= 14)            // Splitting-If
        for (; y < 49; y++)             // Second y-loop
          for (vx = 0; vx < 9; vx++) ... {    // No
            then_block_1; then_block_2; }    // If-Stmts
      else
        for (vx = 0; vx < 9; vx++) ... {
          if (0 || 4*x+x4>35 || 0 || 4*y+y4>48)   // Old
            then_block_1; else else_block_1; // If-Stmts
          if (4*x+vx+x4-4<0 || 4*x+vx+x4-4>35 || ...
            then_block_2; else else_block_2; }
```

Optimized Code Structure

Splitting-If

– Whenever the condition of the splitting-if is satisfied, the conditions of all original if-statements are automatically also satisfied.

☞ Then-part of the splitting-if thus contains no original if-statements any more, but only their then-parts.

– If splitting-if is not satisfied, no safe statement about the conditions of the original if-statements is possible.

☞ Else-part of the splitting-if contains all original if-statements in order to keep the code correct.

Why Second y-Loop?

Intuitive Code:

```
for (x=0; x<36; x++)
  for (y=0; y<49; y++)
   if (x>=10 || y>=14)
     for (vx=0; vx<9; vx++) ...
```

☞ ***Splitting-If:***
1 execution for
each individual **y** ∈ [14, 48]

```
y = 16
```

Optimized Code:

```
for (x=0; x<36; x++)
  for (y=0; y<49; y++)
   if (x>=10 || y>=14)
     for (; y<49; y++)
       for (vx=0; vx<9; vx++) ...
```

☞ ***Splitting-If:***
1 execution for
all **y** ∈ [14, 48] *altogether*

```
y = 16
```

Phases of Loop Nest Splitting

- ***Condition Satisfiability:*** Finds individual conditions in if-statements that are always or never satisfied.

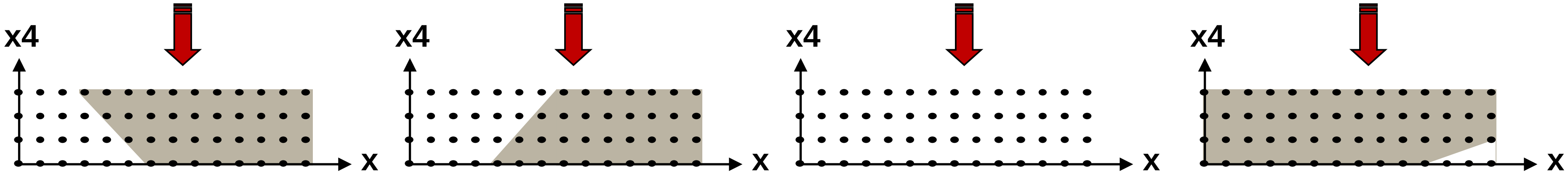
- ***Condition Optimization:*** For each single condition *C*, find a "simpler" condition *C'* such that $C' \Rightarrow C$ holds (whenever *C'* is true, *C* is also true).

- ***Search Space Generation:*** Combine all single conditions *C'* to one data structure *G* that models all if-statements including their structure (`&&`, `||`).

- ***Search Space Exploration:*** Using *G*, determine a condition for the splitting-if that minimizes the number of totally executed if-statements.

Workflow of Loop Nest Splitting (1)

`( 4*x+3*x4>20 && x-x4>3 ) ||` ~~`3*x+x4<0`~~ `|| 6*x-20*x4<61`



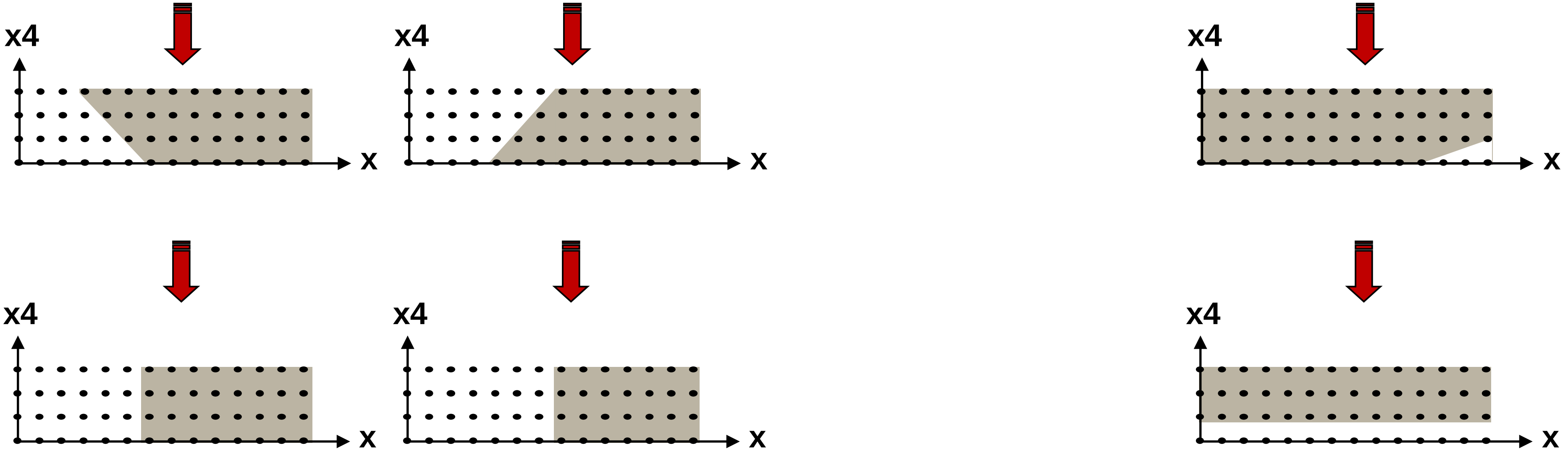
1 - Condition Satisfiability

☞ ***Note:***
This example does not correspond to the MPEG4 code from the previous slides!

– Assumed loop bounds:
$0 \le \mathbf{x} \le 13$
$0 \le \mathbf{x4} \le 3$

Workflow of Loop Nest Splitting (2)

`( 4*x+3*x4>20 && x-x4>3 ) ||` ~~`3*x+x4<0`~~ `|| 6*x-20*x4<61`

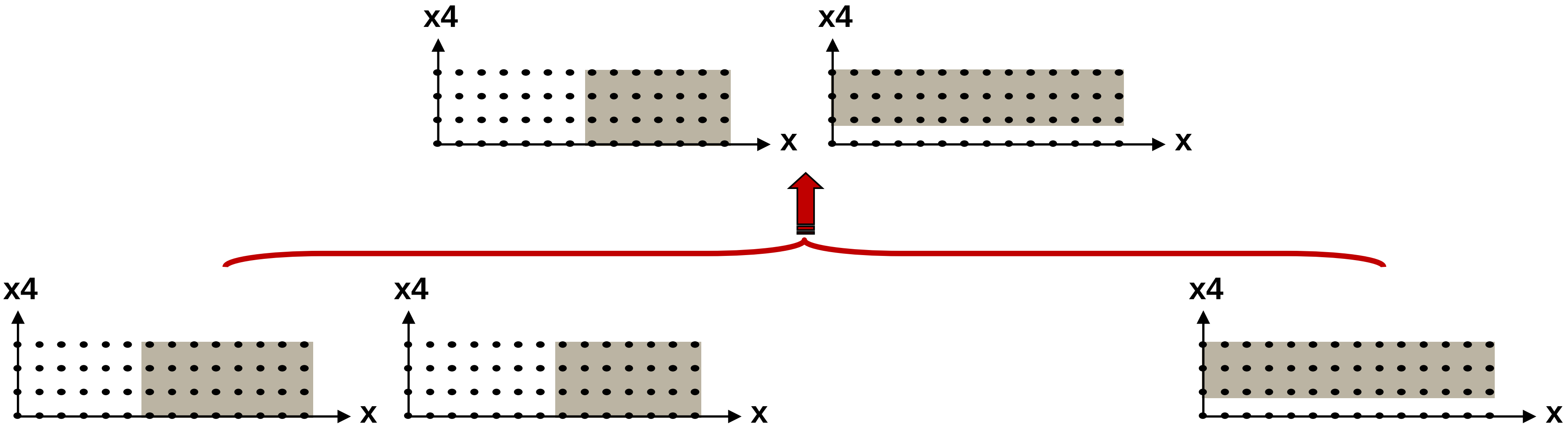


1 - Condition Satisfiability

2 - Condition Optimization

Workflow of Loop Nest Splitting (3)

(4*x+3*x4>20 && x-x4>3) || ~~3*x+x4<0~~ || 6*x-20*x4<61



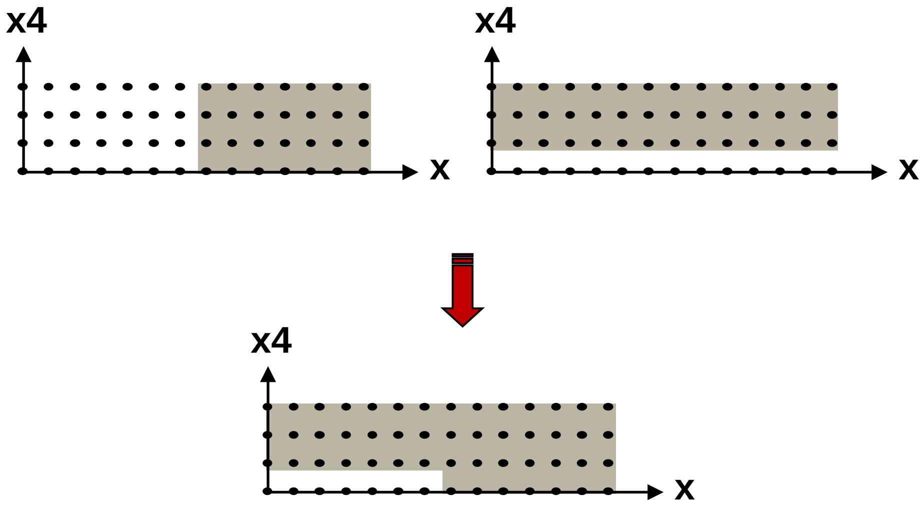
1 - Condition Satisfiability

2 - Condition Optimization

3 - Search Space Generation

Workflow of Loop Nest Splitting (4)

(4*x+3*x4>20 && x-x4>3) || ~~3*x+x4<0~~ || 6*x-20*x4<61



x>=7 || x4>=1

1 - Condition Satisfiability

2 - Condition Optimization

3 - Search Space Generation

4 - Search Space Exploration

Prerequisites

Bounds of Loops L

- All lower and upper bounds (l_L, u_L) are constant

If-Statements

- Sequence of loop-dependent conditions, all of which are combined using logical AND or OR
  - Format: `if (` $C_1 \quad \otimes \quad C_2 \quad \otimes \quad \ldots$ `)` $\qquad \otimes \in \{\&\&, ||\}$

Loop-Dependent Conditions

- Linear expressions over index variables i_L of the loops
  - Format: $C_x \simeq \sum_{L=1}^{N} (c_L * i_L) + c \geq 0 \qquad c_L, c \in \mathbb{Z}$

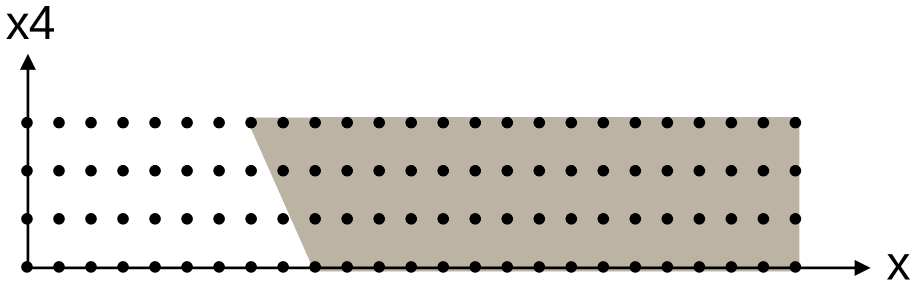
Polytopes & Linear Conditions

Definition (Polyhedra / Polytopes)

- *Polyhedron* $P = \{x \in \mathbb{Z}^N | Ax \geq b\}$ $A \in \mathbb{Z}^{m \times N}, b \in \mathbb{Z}^m$
- Polyhedron P is called *Polytope* iff $|P| < \infty$

Example: Model of Linear Conditions in Nested Loops

- **`4*x + 3*x4 > 35`** for **`x`** $\in [0, 35]$, **`x4`** $\in [0, 3]$ as polytope
- $P = \{p \in \mathbb{Z}^2 | \begin{pmatrix} 4 & 3 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} p \geq \begin{pmatrix} 36 \\ 0 \\ -35 \\ 0 \\ -3 \end{pmatrix}$



Convexity of Polytopes

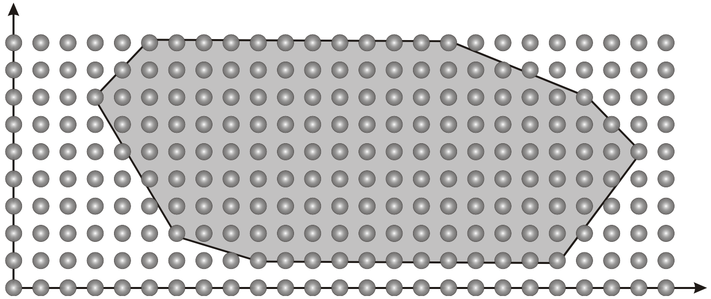
Definition (Convexity)

– A set $S \subset \mathbb{Z}^N$ is called *convex* if each convex combination $ax + by$ of $x, y \in S$ is in S and $(a + b = 1; b \geq 0)$.

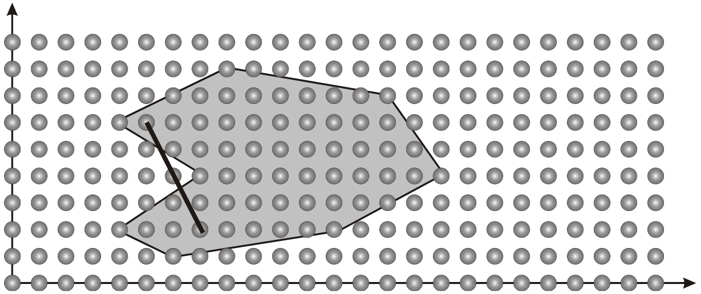
Less Formally

– Each line between any two arbitrary points x and y from S must lie completely in S.

☞ *Polytopes are convex.*



Convex



Not convex

Operations on Polytopes

Properties

- The ***Intersection*** of two polytopes is again a polytope.
- The ***Union*** of two polytopes is not a polytope, since the resulting set is not necessarily convex.

Definition (Finite Union of Polyhedra, FUP)

- For polyhedra $P_1, \ldots, P_n (n \in \mathbb{N}, n < \infty)$, $V := \{P_1 \cup \cdots \cup P_n\}$ is called a *Finite Union of Polyhedra (FUP)*.

Set Operations on FUPs $A = \bigcup_i A_i, B = \bigcup_j B_j$

- $A \cap B := (\bigcup_i A_i) \cap (\bigcup_j B_j) = \bigcup_{i,j} (A_i \cap B_j)$
- $A \cup B := (\bigcup_i A_i) \cup (\bigcup_j B_j)$

Phase 1 – Condition Satisfiability

Goal

- To identify loop-dependent conditions C_x that constantly evaluate to ‘true’ or ‘false’, irrespectively of the values of index variables of all surrounding loops.

Approach

- Translate each condition C_x into a polytope P_x (☞ *slide 23*)
- Test for the empty set: $P_x == \emptyset \Rightarrow C_x$ always ‘false’
- Test for the universe: $P_x == U \Rightarrow C_x$ always ‘true’

Source Code Modification

- Replacement of all these constantly true or false conditions in the source code by constants ‘0’ or ‘1’.

Phase 2 – Condition Optimization

Given

- N loops and one condition $C = \sum_{L=1}^{N} (c_L * i_L) + c \geq 0$

Assumed fictive Situation

```
for ( loop $L_1$ )
  ...
    for ( loop $L_N$ )
      if ( $C$ ) ...
```

Assumption: The loops contain only 1 if-statement with only 1 condition. All other if-statements & conditions are quasi suppressed.

Goal: To determine intervals $[l_{C,1}, u_{C,1}] \dots [l_{C,N}, u_{C,N}]$ such that:

- C is satisfied for all loop iterations within these intervals
- Minimization of the number of executions of if-statements after a hypothetical loop nest splitting based on these intervals.

Optimization of a Condition C

$$C = \sum_{L=1}^{N} (c_L * i_L) + c \geq 0$$

- Determination of values $l_{C,L}$ and $u_{C,L}$ for every loop L
- Interpretation: C is satisfied for all $l_{C,L} \leq i_L \leq u_{C,L}$
- Optimization goal:

  All values $l_{C,L}$ and $u_{C,L}$ minimize the total amount of executed if-statements

- Simplification: The linearity of C implies

  either $l_{C,L} = l_L$ or $u_{C,L} = u_L$
- Consequence: Determination of only one value $v_{C,L}$

  Either $[v_{C,L}, u_{C,L}]$ or $[l_{C,L}, v_{C,L}]$ satisfies condition C

Some Definitions...

Given: $C = \sum_{L=1}^{N} (c_L * i_L) + c \geq 0$ $\quad l_L, u_L \quad v_{C,L}$

- **Total Iteration Space** *(#Executions of the innermost loop's body)*

$$TIS = \prod_{L=1}^{N} (u_L - l_L + 1)$$

- **Constrained Iteration Space** *(#Executions of the innermost loop's body, constrained to regions specified by values $v_{C,L}$)*

$$CIS = \prod_{L=1}^{N} r_L$$

$$r_L = \begin{cases} u_L - l_L + 1 & \text{for } c_L = 0, \\ u_L - v_{C,L} + 1 & \text{for } c_L > 0, \\ v_{C,L} - l_L + 1 & \text{otherwise} \end{cases}$$

- **Splitting Loop** *(Index of that loop where splitting will take place)*

$$\lambda = \max\{\ i \mid L_i \in \Lambda, r_L \neq u_L - l_L + 1\ \}$$

Illustration (1)

```
Loop 1       for (i = 0; i < 20; i++)
               for (x = 0; x < 36; x++)
                 for (y = 0; y < 49; y++)
                   for (vx = 0; vx < 9; vx++)
                     for (vy = 0; vy < 9; vy++)
                       for (x4 = 0; x4 < 4; x4++)
Loop N=7                 for (y4 = 0; y4 < 4; y4++) {
                           if (4*x+vx+x4-4>35)
                             then_block_1; else else_block_1; }
```

Note:

– Loops are enumerated from 1 ... *N* from the outermost to the innermost loop.

– Only 1 condition in if-statement instead of the many ones from slide 11!

Illustration (2)

```
Loop 1     for (i = 0; i < 20; i++)
             for (x = 0; x < 36; x++)
               for (y = 0; y < 49; y++)
                 for (vx = 0; vx < 9; vx++)
                   for (vy = 0; vy < 9; vy++)
                     for (x4 = 0; x4 < 4; x4++)
Loop N=7               for (y4 = 0; y4 < 4; y4++) {
                         if (4*x+vx+x4-4>35)
                           then_block_1; else else_block_1; }
```

***TIS*:**

- – Amount of executions of the code within the curly braces.
- – Here: 20 * 36 * 49 * 9 * 9 * 4 * 4 = 45,722,880

Illustration (3)

Loop 1

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    for (y = 0; y < 49; y++)
      for (vx = 0; vx < 9; vx++)
        for (vy = 0; vy < 9; vy++)
          for (x4 = 0; x4 < 4; x4++)
            for (y4 = 0; y4 < 4; y4++) {
              if (4*x+vx+x4-4>35)
                then_block_1; else else_block_1; }
```

Loop

Let $v_{C,1} = 0; v_{C,2} = 10; v_{C,3} = \cdots = 0$ **be given.**

- – Condition C is satisfied for all **x** ≥ 10 and **i**, **y**, ..., **y4** ≥ 0.
- – *CIS*: 20 * (36 – 10) * 49 * 9 * 9 * 4 * 4 = 33,022,080

Illustration (4)

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++) ...
          for (y4 = 0; y4 < 4; y4++) {...}
    else
      for (y = 0; y < 49; y++) ...
          for (y4 = 0; y4 < 4; y4++) {
            if (4*x+vx+x4-4>35) ... }
```

CIS: *Number of executions of the code in curly braces for* **x** ≥ 10*.*

Hypothetical Code for $v_{C,2} = 10$

– Splitting Loop $\lambda = 2$ since splitting-if needs to be placed in 2nd loop.

– Question: How often would *all* if-statements shown here be executed?

Counting of If-Statement Executions

- **#If-Statements after Splitting** *(#Original if's + #Splitting-if's)*

  $$IF_{\text{Total}} = IF_{\text{Orig}} + IF_{\text{Split}}$$

- **#Original if's** *(Total iterations without constrained iterations)*

  $$IF_{\text{Orig}} = TIS - CIS$$

- **#Splitting-if's**

  $$IF_{\text{Split}} = \#\textit{Then-blocks} + \#\textit{Else-blocks} = TB + EB$$

- **#Then-Blocks** *(CIS without iterations of loops inside of λ)*

  $$TB = CIS/(\prod_{L=\lambda+1}^{N} (u_L - l_L + 1) * r_\lambda)$$

- **#Else-Blocks** *(#Original if's without loop iterations inside of λ)*

  $$EB = IF_{\text{Orig}}/\prod_{L=\lambda+1}^{N} (u_L - l_L + 1)$$

Illustration (5)

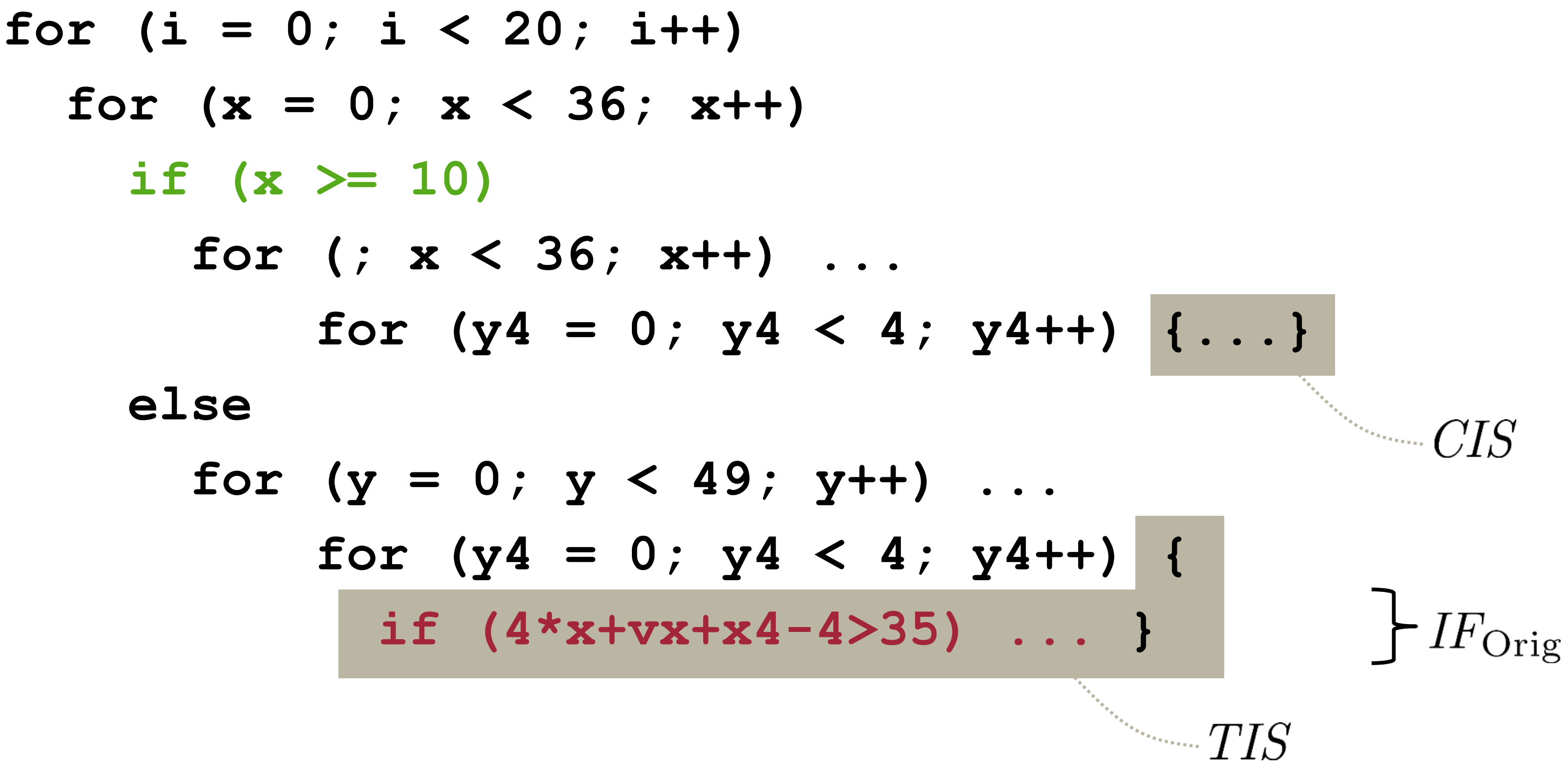
```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++) ...
          for (y4 = 0; y4 < 4; y4++) {...}
    else
      for (y = 0; y < 49; y++) ...
          for (y4 = 0; y4 < 4; y4++) {
            if (4*x+vx+x4-4>35) ... }
```

IF_{Split}

IF_{Orig}

IF_{Total}

Illustration (6)



```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++) ...
          for (y4 = 0; y4 < 4; y4++) {...}
    else
      for (y = 0; y < 49; y++) ...
          for (y4 = 0; y4 < 4; y4++) {
            if (4*x+vx+x4-4>35) ... }
```

$$
\begin{aligned}
IF_{\text{Orig}} &= 20 * 36 * 49 * 9 * 9 * 4 * 4 \; - \\
&\quad 20 * (36 - 10) * 49 * 9 * 9 * 4 * 4 \\
&= 45,722,880 - 33,022,080 = 12,700,800
\end{aligned}
$$

Illustration (7)

```
for (i = 0; i < 20; i++)
 for (x = 0; x < 36; x++)
  if (x >= 10)
   for (; x < 36; x++) ...
     for (y4 = 0; y4 < 4; y4++) {...}
  else
   for (y = 0; y < 49; y++) ...
     for (y4 = 0; y4 < 4; y4++) {
      if (4*x+vx+x4-4>35) ... }
```

$\left.\begin{array}{l} \\ \end{array}\right\} IF_{\text{Split}}$

$\#\mathit{Then\text{-}blocks}$

$\#\mathit{Else\text{-}blocks}$

Illustration (8)

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++) ...
          for (y4 = 0; y4 < 4; y4++) {...}
    else
```

$IF_{\text{Orig}}/4/4/9/9/49$

```
      for (y = 0; y < 49; y++)
```

$IF_{\text{Orig}}/4/4/9/9$

```
        for (vx = 0; vx < 9; vx++)
```

$IF_{\text{Orig}}/4/4/9$

```
          for (vy = 0; vy < 9; vy++)
```

$IF_{\text{Orig}}/4/4$

```
            for (x4 = 0; x4 < 4; x4++)
```

$IF_{\text{Orig}}/4$

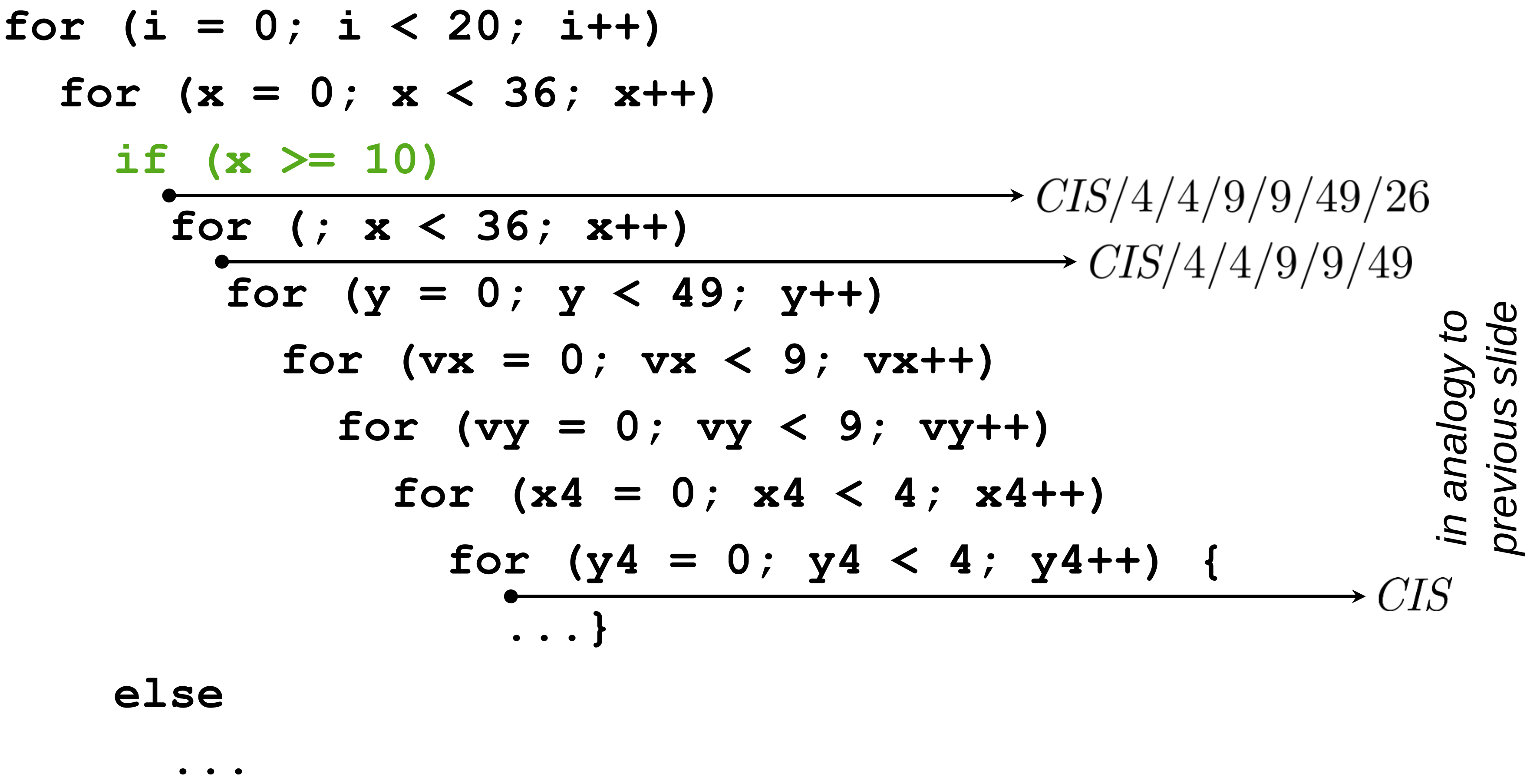
```
              for (y4 = 0; y4 < 4; y4++) {
```

IF_{Orig}

```
                if (4*x+vx+x4-4>35) ... }
```

$$\#\textit{Else-blocks} = IF_{\text{Orig}}/(49 * 9 * 9 * 4 * 4)$$
$$= 12,700,800/63,504 = 200$$

Illustration (9)



```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++)
        for (y = 0; y < 49; y++)
          for (vx = 0; vx < 9; vx++)
            for (vy = 0; vy < 9; vy++)
              for (x4 = 0; x4 < 4; x4++)
                for (y4 = 0; y4 < 4; y4++) {
                  ...}
    else
      ...
```

$CIS/4/4/9/9/49/26$

$CIS/4/4/9/9/49$

CIS

in analogy to previous slide

$$\#\mathit{Then\text{-}blocks} = CIS/(26 * 49 * 9 * 9 * 4 * 4)$$

$$= 33,022,080/1,651,104 = 20$$

Illustration (10)

```
for (i = 0; i < 20; i++)
  for (x = 0; x < 36; x++)
    if (x >= 10)
      for (; x < 36; x++) ...
          for (y4 = 0; y4 < 4; y4++) {...}
    else
      for (y = 0; y < 49; y++) ...
          for (y4 = 0; y4 < 4; y4++) {
            if (4*x+vx+x4-4>35) ... }
```

$\}\ IF_{\text{Split}}$

$\}\ IF_{\text{Orig}}$

$\}\ IF_{\text{Total}}$

$$\begin{aligned} IF_{\text{Total}} &= IF_{\text{Orig}} + IF_{\text{Split}} = IF_{\text{Orig}} + \#\textit{Then-Blocks} + \#\textit{Else-Blocks} \\ &= 12,700,800 + 20 + 200 \\ &= 12,701,020 \end{aligned}$$

Computation of Values $v_{C,L}$

Wrap-Up

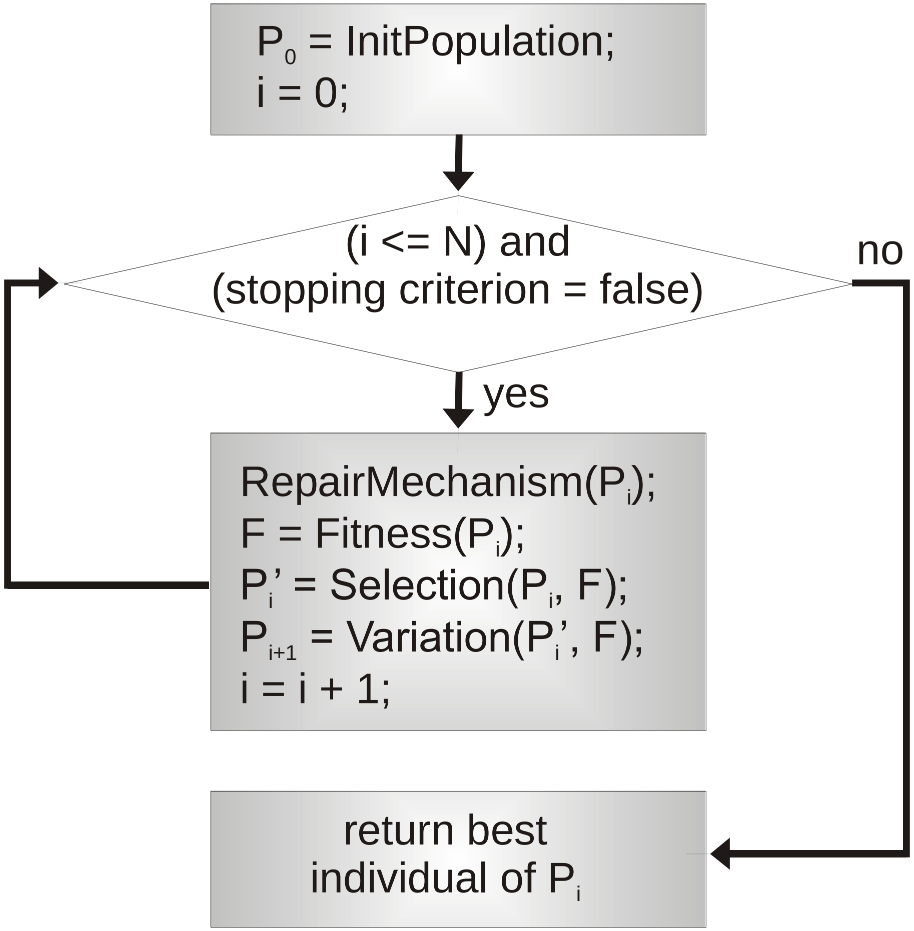
- For a condition C and given values $v_{C,L}$, we can compute how many if-statements would be executed after splitting based on $v_{C,L}$.

Very nice, but...

... who produces good values for $v_{C,L}$?

☞ **A Genetic Algorithm**

Workflow of Genetic Algorithms (1)



P_0 = InitPopulation;
i = 0;

(i <= N) and
(stopping criterion = false)

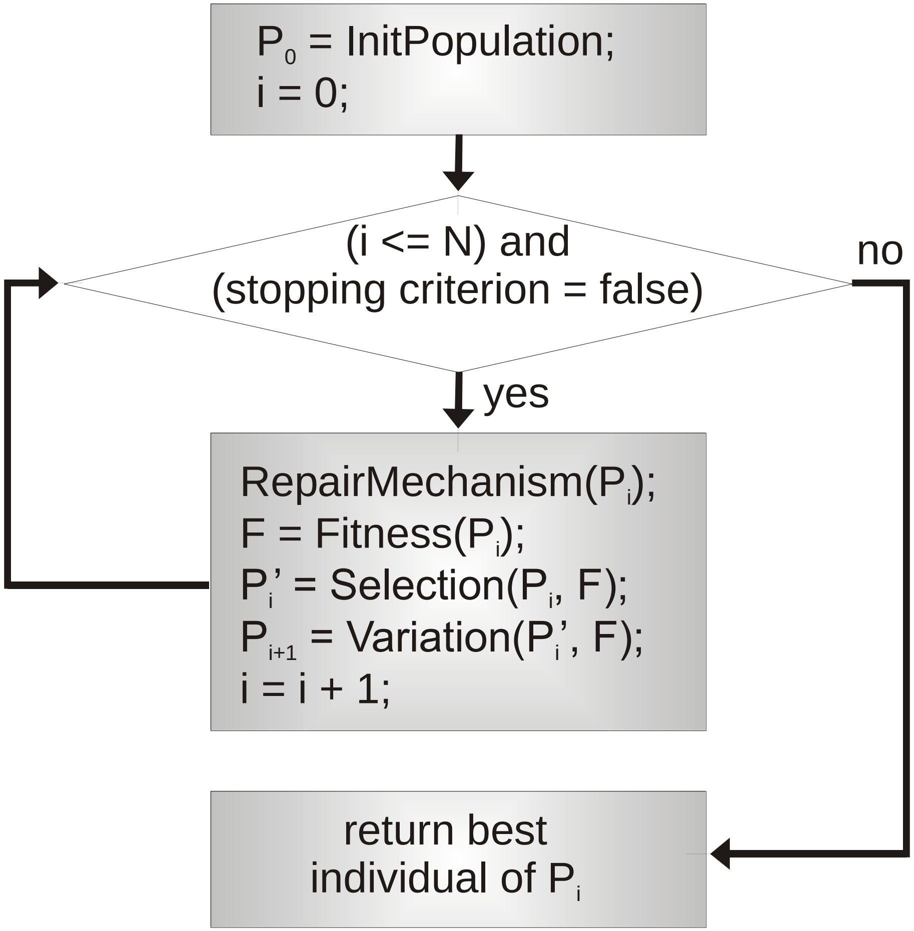
RepairMechanism(P_i);
F = Fitness(P_i);
P_i' = Selection(P_i, F);
P_{i+1} = Variation(P_i', F);
i = i + 1;

return best individual of P_i

- In the style of natural evolution, *“survival of the fittest”*
- Optimization loop *i* = 0, 1, ...
- Each iteration *i* maintains *population* P_i; a population contains several *individuals*
- An individual represents one possible solution for the modeled optimization problem

<INTERLUDE>

Workflow of Genetic Algorithms (2)



P_0 = InitPopulation;
i = 0;

(i <= N) and
(stopping criterion = false)

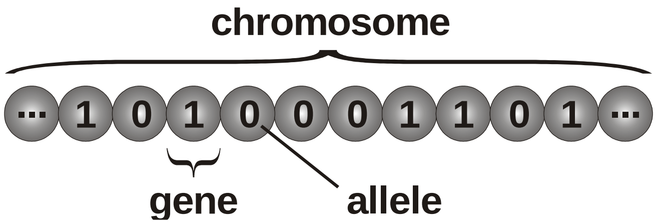
yes

no

RepairMechanism(P_i);
F = Fitness(P_i);
P_i' = Selection(P_i, F);
P_{i+1} = Variation(P_i', F);
i = i + 1;

return best
individual of P_i

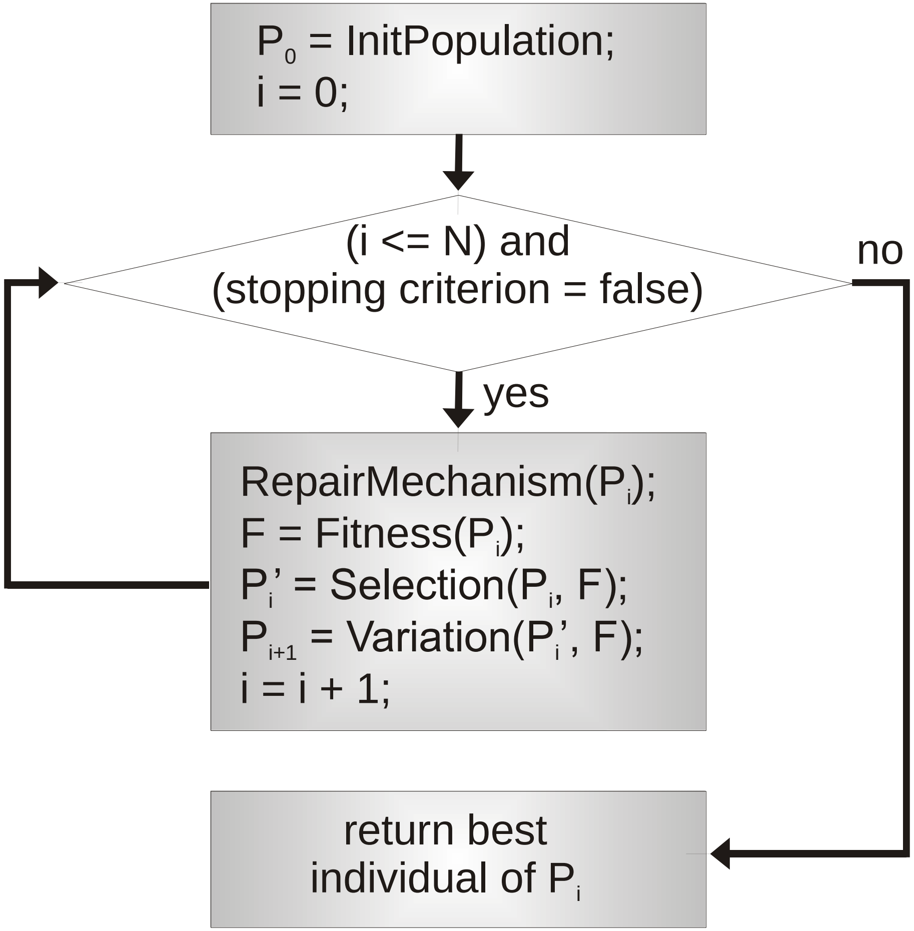
- An individual's data structure is called *chromosome*.



- A chromosome consists of many *genes* that are used to save data.
- One actual value stored in a gene is called *allele*.

<INTERLUDE>

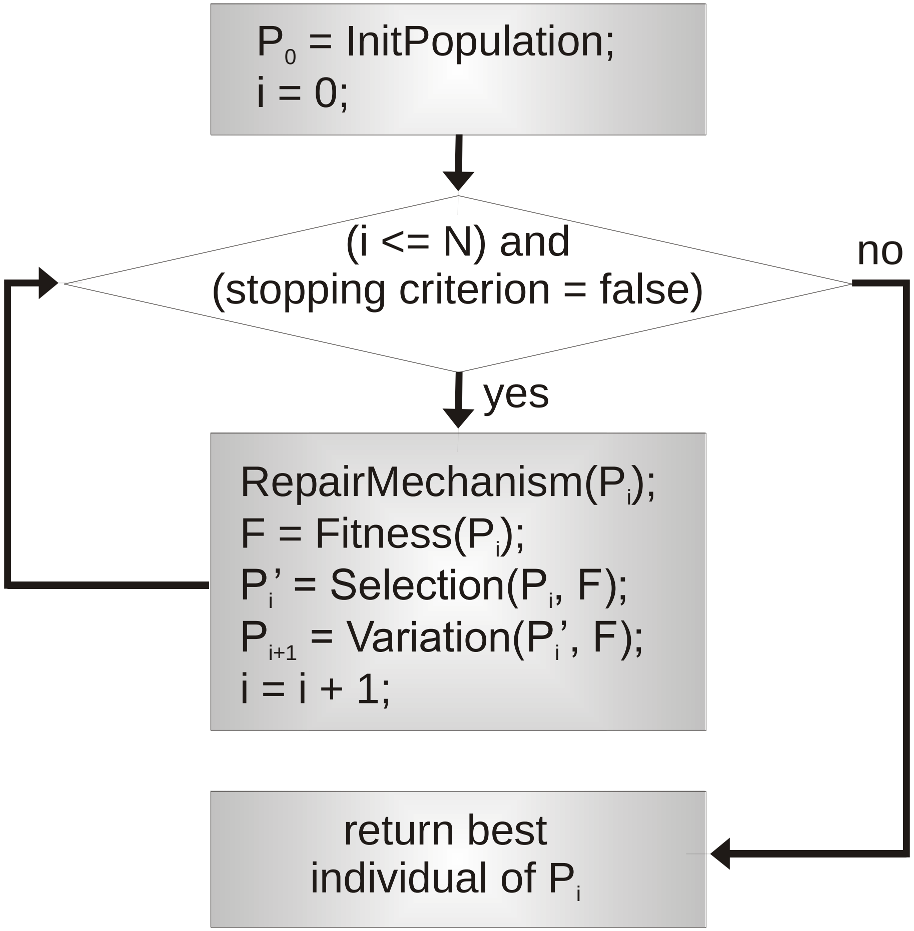
Workflow of Genetic Algorithms (3)



- A fitness function computes the *fitness* of each individual inside P_i.
- From P_i, a subset P_i' of highest / lowest fitness is selected (*selection*, depending on whether a minimization or maximization problem is optimized).
- P_i' is completed to the next population P_{i+1} by randomly generating new individuals *(variation)*

<INTERLUDE>

Workflow of Genetic Algorithms (4)



P_0 = InitPopulation;
i = 0;

(i <= N) and
(stopping criterion = false)

no

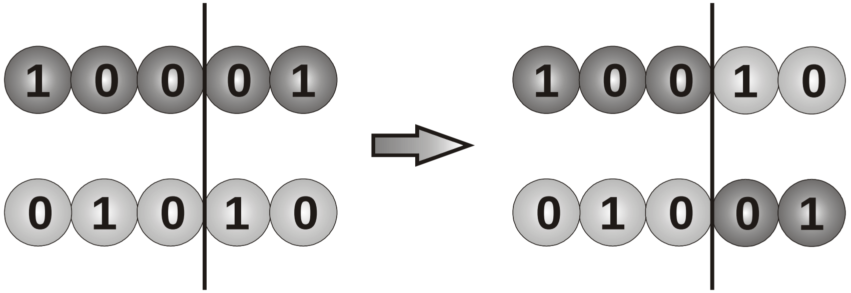
yes

RepairMechanism(P_i);
F = Fitness(P_i);
P_i’ = Selection(P_i, F);
P_{i+1} = Variation(P_i’, F);
i = i + 1;

return best
individual of P_i

– Variation makes use of two basic genetic operators:

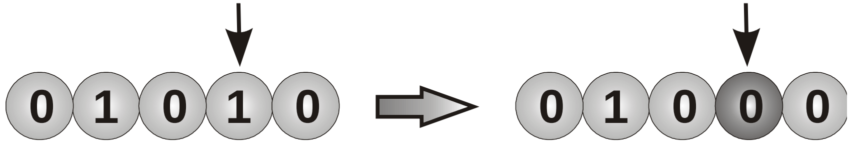
– *Cross-over:*



1 0 0 0 1 → 1 0 0 1 0

0 1 0 1 0 → 0 1 0 0 1

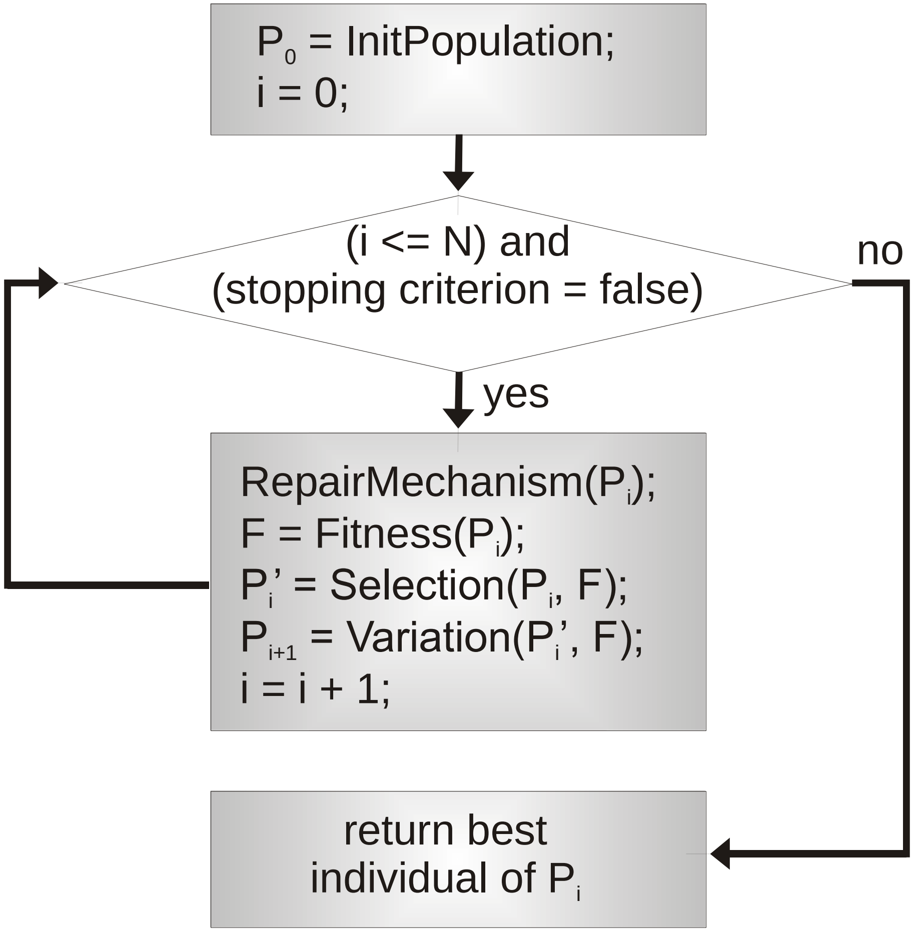
– *Mutation:*



0 1 0 1 0 → 0 1 0 0 0

<INTERLUDE>

Workflow of Genetic Algorithms (5)



P_0 = InitPopulation;
i = 0;

(i <= N) and
(stopping criterion = false)

yes

no

RepairMechanism(P_i);
F = Fitness(P_i);
P_i' = Selection(P_i, F);
P_{i+1} = Variation(P_i', F);
i = i + 1;

return best
individual of P_i

- Due to the randomness in variation, P_i can contain individuals that do not represent valid solutions: *Repair* mechanism.
- Termination of the GA if
  - max. *N* iterations reached,
  - best observed fitness unchanged for *y* iterations,
  - ...
- Final result is that individual from last population with best fitness.

</INTERLUDE>

Genetic Algorithm for Condition Optimization

Chromosomal Representation

- For *N* nested loops, each chromosome has *N* genes
- Each gene holds one integer number
- Gene *L* represents the value $v_{C,L}$ to be optimized
- Domain of each gene *L* restricted to interval $[l_L, u_L]$

Fitness

- Fitness of an individual = IF_{Total}
- Invalid individuals $(v_{C,1}, \ldots, v_{C,N})$ represent iterations within the loop nest in which condition C is not always satisfied
- Invalid individuals obtain a very poor fitness

Result of Condition Optimization

Inputs to Condition Optimization

- Linear condition C
- Loop bounds $[l_L, u_L]$

Output of the Genetic Algorithm

- Values $(v_{C,1}, \ldots, v_{C,N})$ of the individual with best fitness

Output of Condition Optimization Phase

- Polytope

$$P'_C = \left\{ (x_1, \ldots, x_N) \in \mathbb{Z}^N \mid \forall \text{ Loops } L : \begin{array}{l} l_L \leq x_L \leq u_L, \\ x_L \geq v_{C,L} \text{ if } c_L > 0, \\ x_L \leq v_{C,L} \text{ if } c_L < 0 \end{array} \right\}$$

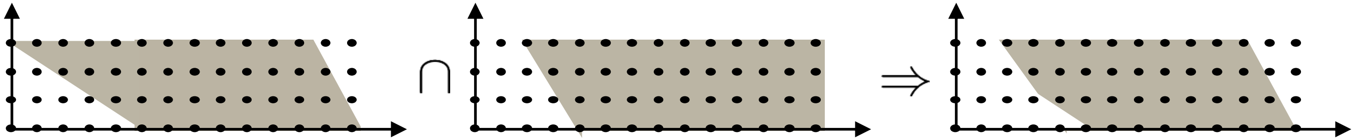
Phase 3 – Search Space Generation

Given: If-statements, conditions & polytopes

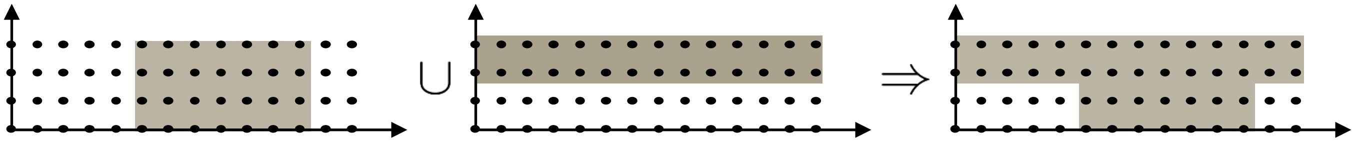
$$\mathrm{IF}_i = (C_{i,1} \otimes C_{i,2} \otimes \cdots \otimes C_{i,n}), \otimes \in \{\&\&, \,|\,|\} \qquad \forall C_{i,j} \rightsquigarrow P'_{i,j}$$

Construction of FUPs *P*$_i$ for each full if-statement IF$_i$

- $\text{if } C_{i,j-1} \;\; \&\& \;\; C_{i,j} :$



- $\text{if } C_{i,j-1} \;\; |\,| \;\; C_{i,j} :$



Construction of a Global FUP *(Global Search Space)*

FUP *G* models iteration space in which *all* if-statements are satisfied.
Constructed by intersecting all FUPs *P*$_i$ of the individual if-statements:

- $G = \bigcap P_i$

Phase 3 – Structure of the FUP *G*

Consequence of Using the $\cup$ Operator on the Previous Slide

- *G* is a finite union of polytopes and can thus be seen as:
- $G = R_1 \cup R_2 \cup \cdots \cup R_M$
- Interpretation:

  Each polytope R_r denotes

  *one region in the iteration space of the entire loop nest in which **all** if-statements are satisfied.*

Global Search Space for MPEG Code

- $\mathrm{IF}_1 = \texttt{4*x+x4<0 || 4*x+x4>35 || 4*y+y4<0 || 4*y+y4>48}$

  $P_{1,1} = \emptyset, P_{1,2} = \{\mathrm{x} \geq 9\}, P_{1,3} = \emptyset, P_{1,4} = \{\mathrm{y} \geq 13\}$

  $P_1 = \{\mathrm{x} \geq 9\} \cup \{\mathrm{y} \geq 13\}$

- $\mathrm{IF}_2 = \texttt{4*x+vx+x4<4 || 4*x+vx+x4>39 || 4*y+vy+y4<4 || 4*y+vy+y4>52}$

  $P_{2,1} = \{\mathrm{x} = 0 \ \wedge \ \mathrm{vx} = 0\}, P_{2,2} = \{\mathrm{x} \geq 10\},$

  $P_{2,3} = \{\mathrm{y} = 0 \ \wedge \ \mathrm{vy} = 0\}, P_{2,4} = \{\mathrm{y} \geq 14\}$

  $P_2 = \{\mathrm{x} = 0 \ \wedge \ \mathrm{vx} = 0\} \cup \{\mathrm{x} \geq 10\} \cup \{\mathrm{y} = 0 \ \wedge \ \mathrm{vy} = 0\} \cup \{\mathrm{y} \geq 14\}$

- $G = P_1 \cap P_2 =$

  $$\{\mathrm{x} = 0 \ \wedge \ \mathrm{vx} = 0 \ \wedge \ \mathrm{y} \geq 13\} \ \cup \ \{\mathrm{x} \geq 10\} \ \cup$$

  $$\{\mathrm{y} = 0 \ \wedge \ \mathrm{vy} = 0 \ \wedge \ \mathrm{x} \geq 9\} \ \cup \ \{\mathrm{y} \geq 14\}$$

Global Search Space & Splitting-If (1)

- $G = \mathrm{IF}_1 \cap \mathrm{IF}_2 =$

$$\{\mathrm{x} = 0 \ \wedge \ \mathrm{vx} = 0 \ \wedge \ \mathrm{y} \geq 13\} \ \cup \ \{\mathrm{x} \geq 10\} \cup$$
$$\{\mathrm{y} = 0 \ \wedge \ \mathrm{vy} = 0 \ \wedge \ \mathrm{x} \geq 9\} \ \cup \ \{\mathrm{y} \geq 14\}$$

Direct Translation of *G* into Splitting-If

```
if ( (x == 0 && vx == 0 && y >= 13) || (x >= 10) ||
     (y == 0 && vy == 0 && x >= 9) || (y >= 14) )
```

Not a Good Idea

☞ This splitting-if must be placed in **`vy`**-loop (3rd-innermost!)

☞ Leads to 10,103,760 executions of if-statements in total

Global Search Space & Splitting-If (2)

- $G = \mathrm{IF}_1 \cap \mathrm{IF}_2 =$

$$\{\mathrm{x} = 0 \ \wedge \ \mathrm{vx} = 0 \ \wedge \ \mathrm{y} \geq 13\} \ \cup \ \{\mathrm{x} \geq 10\} \cup$$
$$\{\mathrm{y} = 0 \ \wedge \ \mathrm{vy} = 0 \ \wedge \ \mathrm{x} \geq 9\} \ \cup \ \{\mathrm{y} \geq 14\}$$

Alternative: Use only sub-polytopes R_r of *G* for splitting-if.
Legal, since each sub-polytope R_r by itself already satisfies all if-statements.

```
if ( x >= 10 )
```

- This sub-polytope is also not a good solution
  - ☞Leads to 25,401,820 if-statement executions

```
if ( ( x >= 10 ) || ( y >= 14 ) )
```

- This combination of sub-polytopes is a good solution
  - ☞Leads to 7,261,120 if-statement executions

Phase 4 – Search Space Exploration

GA: Selects regions from $G = R_1 \cup R_2 \cup \dots \cup R_M$

- Individual: Bit-vector that marks the selected regions

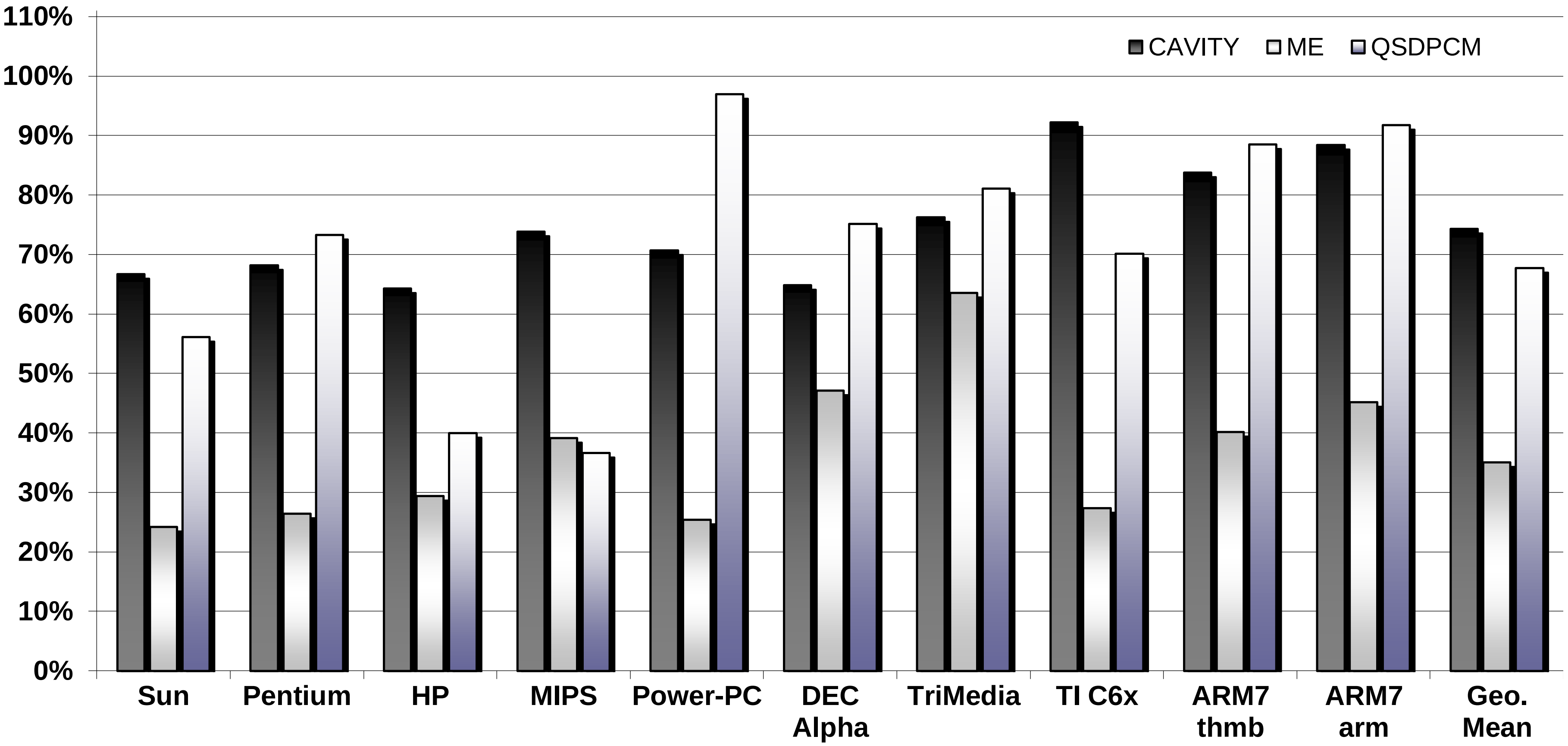
  Individual $I = (I_1, \dots, I_M), I_r = \begin{cases} 1 & \text{if region } R_r \text{ selected,} \\ 0 & \text{otherwise} \end{cases}$

- Fitness function computes again #{If-statement executions} after loop nest splitting
- Fitness function minimized by GA

Resulting Splitting-If

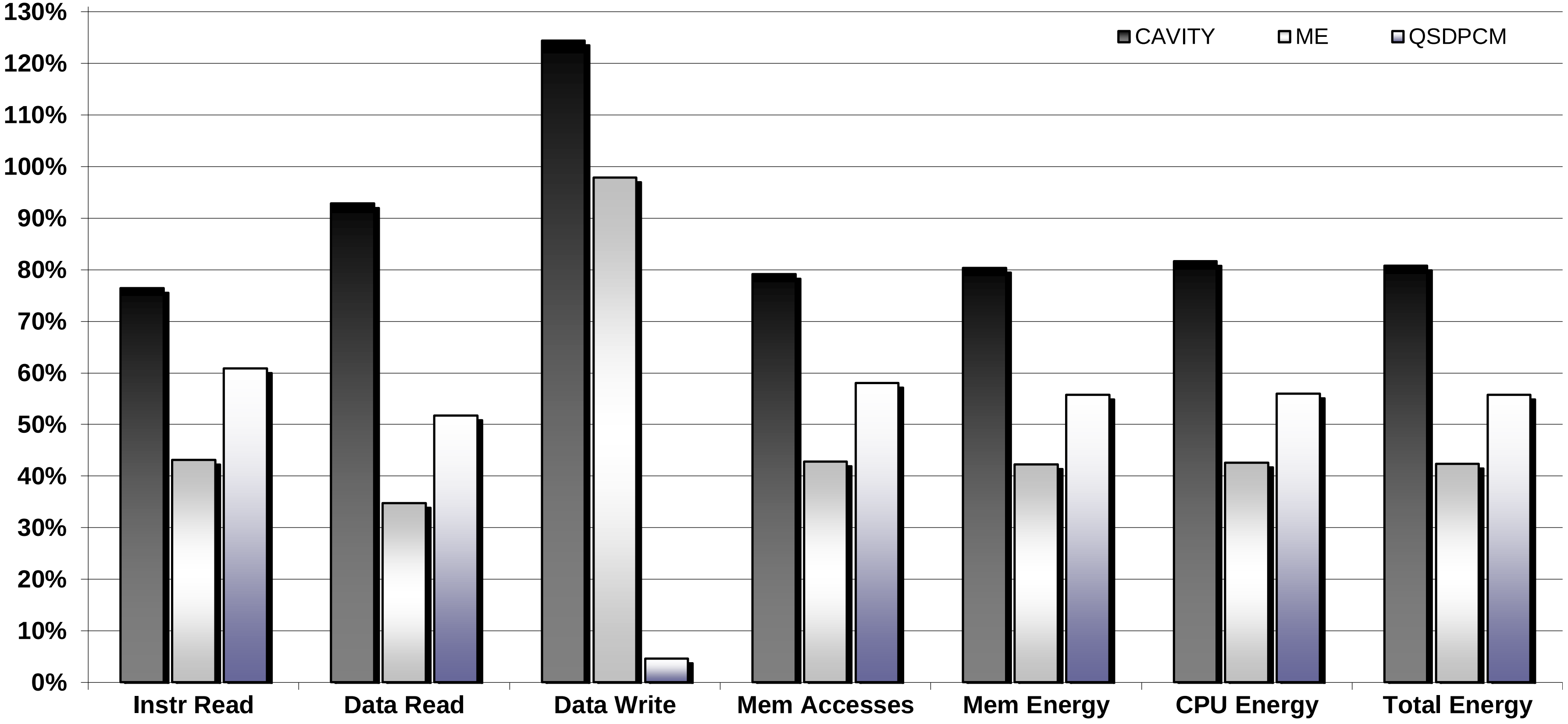
- Placed in the outermost possible loop of the loop nest
- Contains all conditions and operators as specified by the selected regions R_r

Relative Run-Times after Loop Nest Splitting



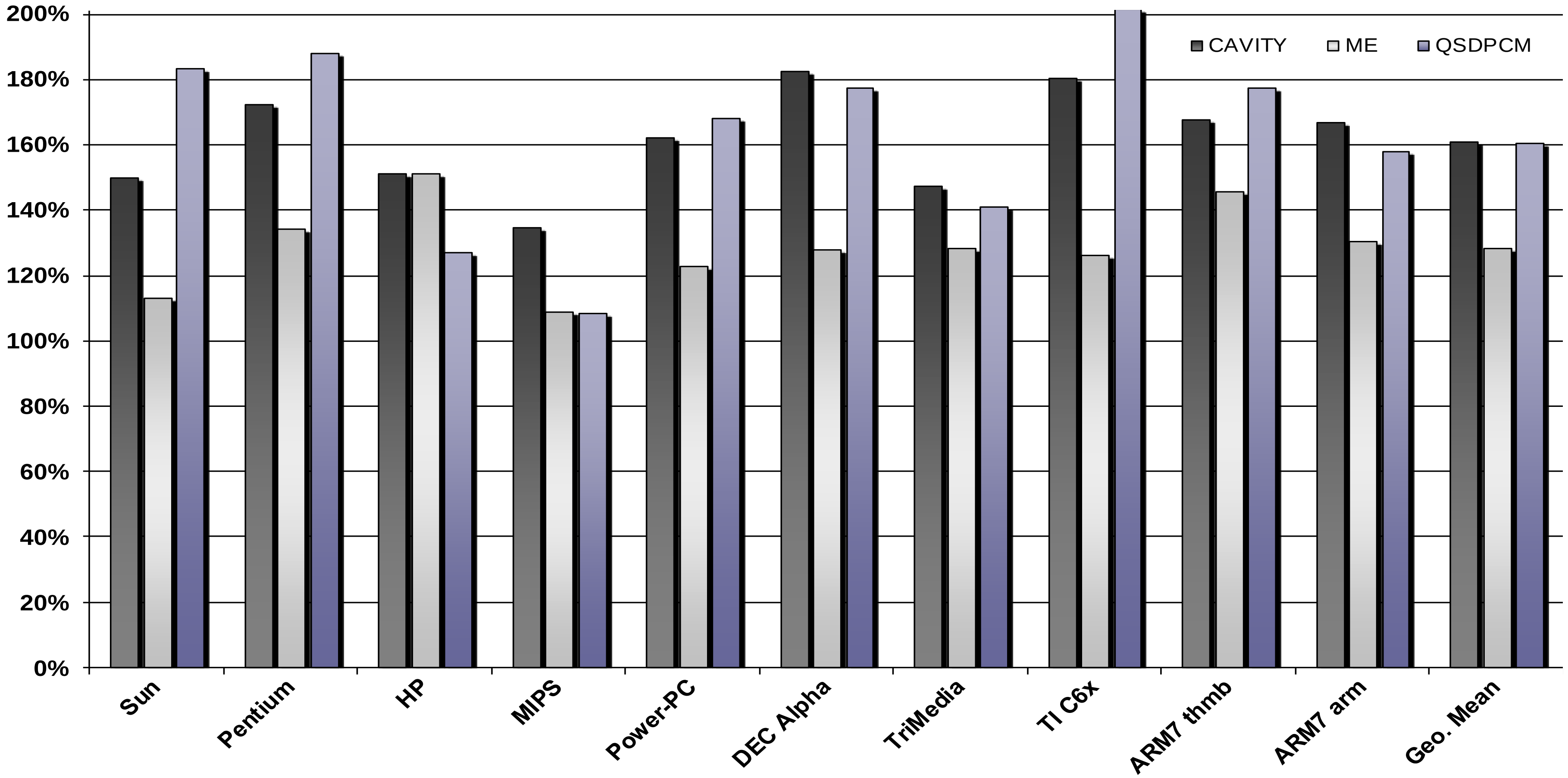
100% = Run-times of the benchmarks without Loop Nest Splitting

Relative Energy Consumption (ARM7) after LNS



100% = Values of the Benchmarks without Loop Nest Splitting

Relative Code Size after Loop Nest Splitting



100% = Size of the benchmarks without Loop Nest Splitting

References

Loop Nest Splitting

– H. Falk, P. Marwedel. *Control Flow driven Splitting of Loop Nests at the Source Code Level*. DATE Conference, Munich, 2003.

– H. Falk. *Control Flow Optimization by Loop Nest Splitting at the Source Code Level*. University of Dortmund, Technical Report No. 773, Dortmund 2003.

Summary

Non-Compiler Optimizations

- *Post-pass* after compiler, e.g., at linker-level
- *Pre-pass* before compiler, at source code level

Loop Nest Splitting

- Control flow optimization in data flow-dominated multimedia applications
- Polytopes used to model linear conditions and loops
- Genetic algorithms used to optimize polytope models
- Significant reductions in terms of ACET and energy *(and WCET)*, but partially heavy code size increases