Computer Graphics

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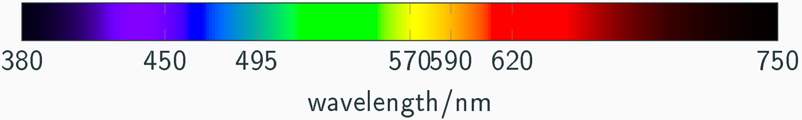
Light & Color

Photons



- Light can be thought of as particle or wave
- In computer graphics we use the quantum nature of light
- Each quantum is referred to as **photons**
- Each has an energy of

$$E = \frac{hc}{\lambda} \quad (1)$$

where h is the Planck constant and c is the speed of light.

Spectral Energy

- High number of photons are impractical to handle
- E.g. computation of the total energy requires summation over all photons
- However this information is inaccessible by measurements

Definition
To describe how energy is distributed across wavelength one can only measure the energy ΔE of photons in certain interval $\Delta\lambda = \lambda_2 - \lambda_1$, $\lambda_1 < \lambda_2$ and define the spectral energy by

$$Q_\lambda = \frac{Q_\lambda}{\Delta t} = \frac{\Delta E}{\Delta \lambda}, \tag{2}$$

where we think of $\Delta\lambda$ being small, but large enough that the quantum nature of light does not come into play.

Spectral Power

Definition
To describe the rate of energy production we define the spectral power as the spectral energy emitted per time interval $\left[t - \frac{\Delta t}{2}, t + \frac{\Delta t}{2}\right]$ of length Δt

$$\Phi_\lambda = \frac{\Delta E}{\Delta t \Delta \lambda}. \tag{3}$$

Example
Laser pointer ($\lambda = 532\,\text{nm}$, $\Phi_\lambda \approx 1\,\text{mW}$) emits around $2e15$ photons per second

Color - Definition

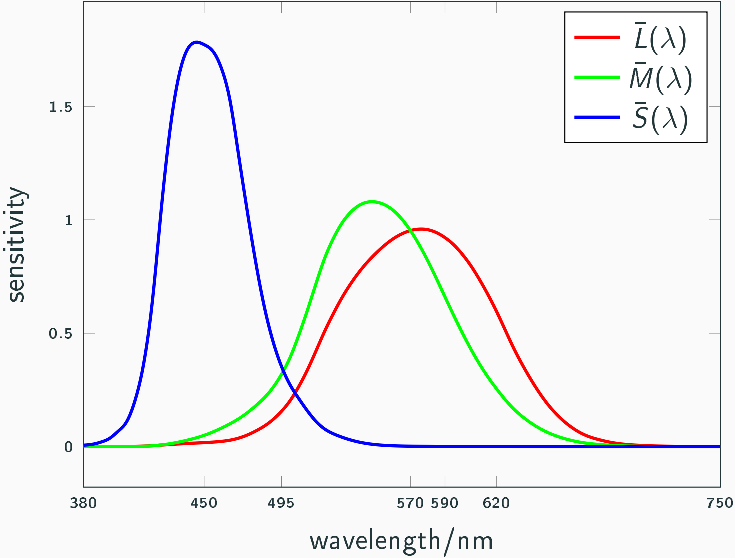
Definition
Color is the aspect of visual perception by which an observer may distinguish differences between two structure-free fields of view of the same shape and size, such as may be caused by differences in the spectral composition of the radiant energy concerned in the observation.

Wyszecki & Stiles, *Color Science: Concepts and Methods, Quantitative Data and Formulae* (Second ed.), New York: Wiley, 2000.

Visual Sensors

- Human eye has two types of photodetectors: rods and cones
- highly sensitive rods for low-light conditions
- three types of cones S, M, and L for normal light (daylight) conditions
    - S: short wavelength receptor
    - M: medium wavelength receptor
    - L: long wavelength receptor

Cone Responses



Tristimulus Value

Definition

The response to a stimulus with spectral composition Φ is given by the three integrated responses

$$
\begin{aligned}
L &= \int_0^\infty \Phi(\lambda)\bar{L}(\lambda)\mathrm{d}\lambda \\
M &= \int_0^\infty \Phi(\lambda)\bar{M}(\lambda)\mathrm{d}\lambda \\
S &= \int_0^\infty \Phi(\lambda)\bar{S}(\lambda)\mathrm{d}\lambda,
\end{aligned}
$$

where $\bar{L}$, $\bar{M}$, and $\bar{S}$ (sometimes without bar) are the cone response functions. We refer to these responses as **tristimulus values**.

Note

It is possible to find spectral compositions $\Phi_1 \neq \Phi_2$ yielding the same tristimulus values. One refers to this phenomena to as **metamerism**, which is the basis of all color reproduction devices.

Grassmann's Laws

Describe emirically the perception of mixtures of colored lights.

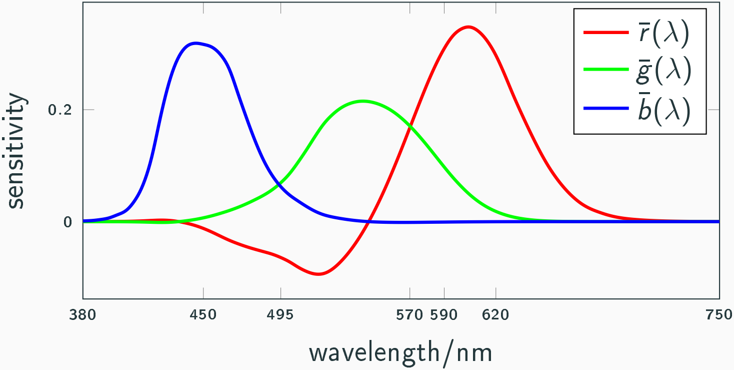
- **Symmetry law:** If color stimulus A matches color stimulus B, then B matches A
- **Transitive law:** If A matches B and B matches C, then A matches C
- **Proportionality law:** If A matches B, then αA matches αB, $\alpha \geq 0$
- **Additivity law:** If A matches B, C matches D, and $A + C$ matches $B + D$, then $A + D$ matches $B + C$

Color Matching Experiments

Color matching
Consider three primary light sources, i.e. light sources of different color each with a dial to alter its intensity. Given an unknown light source we may be able to adjust the dials, such that the additive light of the primaries match the percieved color of the unknown source. The position of the dials is then essentially a representation of the fourth light source, though the color spectras are most certainly different.

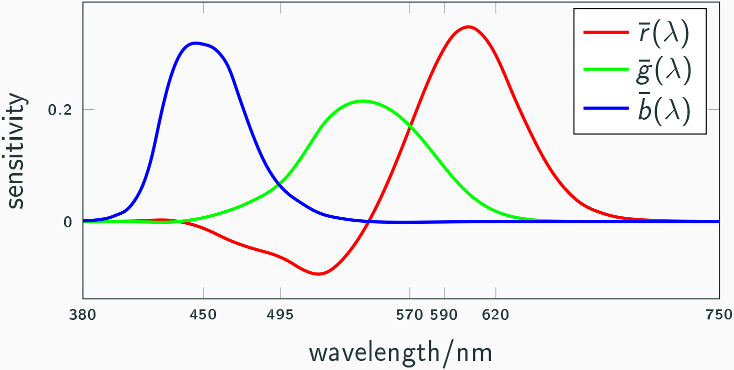
- Results depend on observer
- For color matching no knowledge of the cone response functions is required
- However having to repeat this experiment for a large number of colors would be quite troublesome

Commision Internationale d'Eclairage (CIE) primaries
Using monochromatic light sources of 435.8 nm, 546.1 nm and 700 nm wavelength as primaries, the tristimulus values averaged over many observers look as follows



RGB Color Matching Functions

- Notice the anomaly, where $\bar{r}(\lambda)$ gets negative
- Whenever this happens the color of the source cannot be reproduced by the red, green and blue primaries
- Instead light has to be added to the source to create a match



CIE RGB

RGB Tristimulus values
Using the color matching functions $\bar{r}(\lambda)$ $\bar{g}(\lambda)$ and $\bar{b}(\lambda)$ one can calculate the tristimulus values

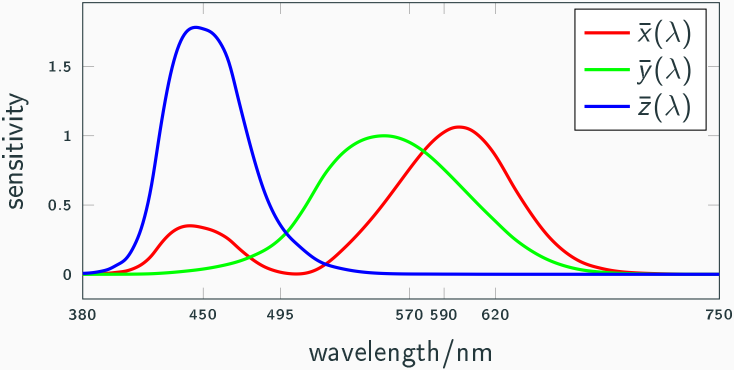
$$R = \int_0^\infty \Phi(\lambda)\bar{r}(\lambda)\mathrm{d}\lambda$$
$$G = \int_0^\infty \Phi(\lambda)\bar{g}(\lambda)\mathrm{d}\lambda$$
$$B = \int_0^\infty \Phi(\lambda)\bar{b}(\lambda)\mathrm{d}\lambda$$

Standard Observer

- The problem with the negative stimuli remains for all physical realizable light sources
- Using imaginary light sources positive color matching functions can be defined

CIE 1931 standard observer
These functions are labeled by $\bar{x}(\lambda)$ $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$, where $\bar{y}$ is equal to the photoptic luminance response function which describes the spectral sensitivity of human visual perception to brightness.

Standard Observer



- Every color can be represented by a set of three positive tristimulus values (X, Y, Z) in a 3D color space.
- The set of all possible tristimulus values is called **color gamut**.

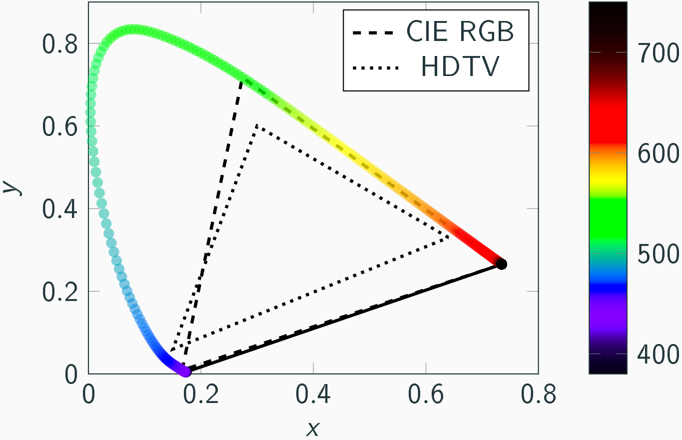
Chromaticity Diagram

Definition
Let

$$x = \frac{X}{X+Y+Z}$$
$$y = \frac{Y}{X+Y+Z}$$
$$z = \frac{Z}{X+Y+Z} \tag{4}$$

be the normalized tristimulus values. The tuple (x, y) projects from the CIE XYZ color space to a 2D space called chromaticity diagram ($x + y + z = 1!$).

Spectrum Inside the Chromaticity Diagram



- Plotting the spectrum in a chromaticity diagram one obtains a horseshoe-shaped curve.
- All other colors are mixtures of the spectrum loci, i.e. they lie within the convex hull of the horse shoe

- The CIE XYZ color space is device independent
- All other color spaces are usually defined by their relationship to CIE XYZ

Example

Consider a trichromatic additive display device and measure the (x, y) coordinates of its primaries $(1, 0, 0)$ (R), $(0, 1, 0)$ (G) and $(0, 0, 1)$ (B) as well as the (X_W, Y_W, Z_W) coordinates of its white point $(1, 1, 1)$. Solve the following linear system

$$\begin{pmatrix} X_W \\ Y_W \\ Z_W \end{pmatrix} = \begin{pmatrix} x_R & x_G & x_B \\ y_R & y_G & y_B \\ z_R & z_G & z_B \end{pmatrix} \begin{pmatrix} S_R \\ S_G \\ S_B \end{pmatrix}. \tag{5}$$

Example
The conversion between RGB and XYZ is then given by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x_R S_R & x_G S_G & x_B S_B \\ y_R S_R & y_G S_G & y_B S_B \\ z_R S_R & z_G S_G & z_B S_B \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}. \quad (6)$$

- If white point and primaries are unknown assume standard RGB (sRGB) encoding (ITU-R BT.709, HDTV encoding)

	R	G	B	White
x	0.64	0.3	0.15	0.3127
y	0.33	0.6	0.06	0.329

- Most transformations contain an additional non-linear transformation to reduce quantization errors in digital applications

Adaptation

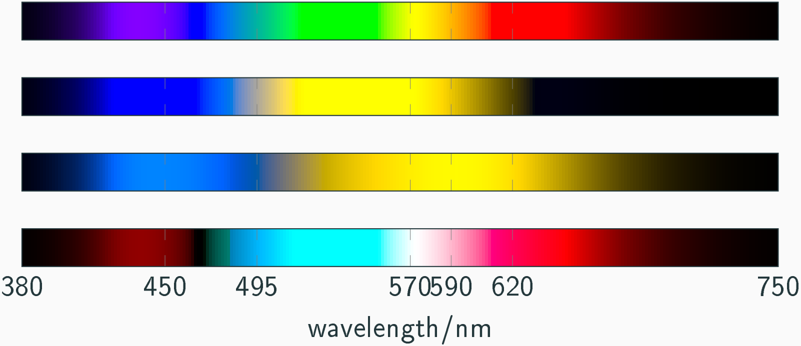
- The context in which we observe objects has a large influence on our perception but is not accounted for here
- Naturally, lighting conditions vary strongly (color and luminance)
- Human vision adapts to those changes allowing us to largely ignore changes in the viewing environment, e.g. white objects appear white for a large variety of lighting conditions
  - light adaption
  - dark adaption
  - chromatic adaption
- Adaptation effects can be modeled to some degree allowing to find the correct stimulus under one lighting condition to match the stimulus under another

Color Blindness

- Deceased ability to see color or differences thereof
- Most common form is red-green color blindness (8% male and 0.5% female Northern Europeans)
- Monocromacy: unable to distinguish colors
- Dichromacy: One of the three cones is absent or missing
- Anomalous trichromacy: altered spectral sensitivity of one or more cone types

Color Blindness - Dichromacy

- Protanopia: lacking of L cones
- Deutanopia: lacking of M cones
- Tritanopia: lacking of S cones



Summary

- Colorimetry allows to describe colors in a device independent manner using the tristimulus values XYZ
- Each display device has its own color space
- They are not able to display all colors (negative tristimulus values), but only a device specific subset (color gamut)
- We can transform between color spaces by a combination of linear and non-linear transformations
- These tools can not be used to describe how colors are perceived, as this depends on the context, in which the color is seen
- If you chose the color of your paint using a digital display it most certainly looks very different in real life