Computer Graphics

Dr. rer. nat. Martin Möddel

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Institut für Biomedizinische Bildgebung

Ray Tracing

Ray Tracing

Figure 1: "Image d'objets transparents montrant les capacités du logiciel POV-Ray." by Gilles Tran released into the public domain.

Definition

A ray tracer is an IOR algorithm, where a ray is traced into a scene until it hits a scene object. Information on hit point and object are used to color image pixels. A basic ray tracer has three parts

1. Ray generation: compute origin and direction of pixels viewing ray based on camera geometry and orientation
2. Ray intersection: Find closest object intersecting the viewing ray
3. Shading: Compute pixel color based on the result of the ray intersection

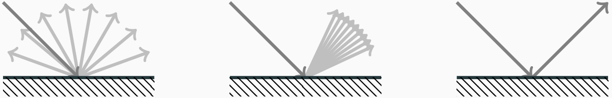
Shading

Definition - Shader
A shader is a program that computes the pixel value once the surface visible to a ray is known using a shader model.

- In ray tracing shader models determine the way light is reflected towards the camera and thus determine how surfaces are iluminated
- The integral part of a shading model is a specific lighting model that determines how light interacts with surfaces
- Realistic lighting models are based on the way light physically interacts with surfaces
- Artistic lighting models create non-realistic looks (Cartoon like)
- In ray tracing most shaders can be implemented by using additional rays cast from the primary hit point and corresponding ray-scene-intersections

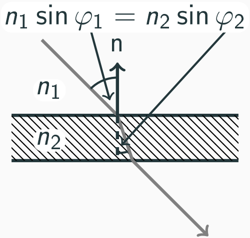
Light Surface Interactions

Reflection



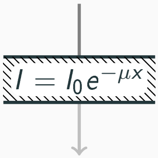
diffuse reflection specular reflection ideal reflection

Refraction

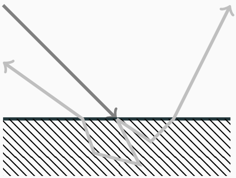


Light Surface Interactions

Absorption



Sub Surface Scattering



Ambient Shading

- The earthly atmosphere scatters light (blue sky and orange sun set)
- Light contribution independent on the surface geometry (normal vector)
- Color of objects might change with color of ambient light
- Often surface color set fixed
- Pixel color L can be described by the surface's ambient coefficient k_a and the intensity of the ambient light I_a

$$L = k_a I_a \tag{1}$$

- Blocking of ambient light due to multiple tightly packed objects not included

Ambient Shading

Figure 2: "View of the complete Mir Space Station after docking against a dark background (015-9,030), over a portion of the Earth (020-2,026-9), and over Australia (023-5). Identifier: STS074-716-021" by NASA released into the public domain

Lambert Shading

- The amount of energy that falls from a light source onto a surface depends on the angle at which the surface is illuminated
- Diffuse and independent of viewing angle
- If n is the unit normal vector of the surface, l is the direction from the hit point to the light source and I is the intensity of the light source at the surface location then

$$L = k_d I \max(0, \mathsf{n} \cdot \mathsf{l}) \tag{2}$$

is the pixel color, where k_d is the surface color or diffuse color

Blin-Phong Shading

- Shiny surfaces produce highlights around the angle of reflection
- Comparing the half vector

$$\mathrm{h} = \frac{-\mathrm{d} + \mathrm{l}}{\| - \mathrm{d} + \mathrm{l}\|_2} \tag{3}$$

  to the normal vector tells us how close v is to the perfect reflection direction
- $\mathrm{n} \cdot \mathrm{h} = 1$ indicates that v is aligned in reflection direction
- $\mathrm{n} \cdot \mathrm{h} = 0$ indicates that v is orthogonal to the reflection direction
- Specular reflection can be modeled by

$$L = k_s I \max(0, \mathrm{n} \cdot \mathrm{h})^p, \tag{4}$$

  where k_s is the specular color, I is the light intensity and p is the so called Phong exponent
- $p = 10$ eggshell, $p = 100$ shiny, and $p = 1000$ glossy

Multiple Light Sources

- Due to light superposition multiple light sources can be implemented by summing up their corresponding contributions

$$L = k_a I_a + \sum_{i=1}^{N} \left(k_d I_i \max(0, \mathrm{n} \cdot \mathrm{l}_i) + k_s I_i \max(0, \mathrm{n} \cdot \mathrm{h}_i)^p \right), \quad (5)$$

  where I_i is the intensity of the i-th light source, l_i is its direction and h_i the corresponding half vector
- The complexity of the ray tracer increases linear with the number of light sources

Luminous intensity

- In CG the term Intensity is usually an alias for luminous intensity
- If the light source is close to the scene such that the distances between light source and illuminated surface vary no fixed intensity values should be used
- Intensity falls off quadratic with the distance to the light source

$$I \propto \frac{1}{d^2} \tag{6}$$

- If the light source is far away from the scene (sun) the variations in intensity are so small that a constant intensity can be used for all objects

Hard Shadows

- So far we assumed either ambient or point like light sources
- The simplistic light models do not create shadows, if occluded by other objects in the scene
- To find out if a surface point p lies in shadow or not, we can create a shadow ray starting at the surface hit by the primary ray in the direction of the light source l

$$\mathrm{p} + s\mathrm{l} \tag{7}$$

- We perform an intersection test
    - If no object is closer to the surface than light source the surface point is lit
    - Else the surface point lies in shadow
- It is important to test for $s \in [\varepsilon, \infty]$, $\varepsilon > 0$ to avoid collision with the surface at the origin
- Performance hit depending on number of light sources

Ideal Reflection

- In case of ideal reflection the color observed by the viewer is not determined by the reflective surface
- Instead one has to consider what a viewer located at the reflective surface looking into the direction of the reflected ray sees, i.e. casting a secondary ray
- This direction can be computed by

$$\mathrm{r} = \mathrm{d} - 2(\mathrm{d} \cdot \mathrm{n})\mathrm{n}, \tag{8}$$

  where n is the normalized surface normal
- The pixel color is determined by the color L_p already collected along the path of the ray and the contribution picked up along the reflection ray

$$L = L_p + \mathrm{raycolor}(\mathrm{p} + s\mathrm{r}) \tag{9}$$

Ideal Reflection

- It is important to test for $s \in [\varepsilon, \infty]$, $\varepsilon > 0$ to avoid collision of the reflective surface and the reflected ray
- To ensure termination of the recursion a maximal recursion depth should be used
- This method does hit performance significantly by introducing jet more intersections into the programm
- for efficiency no reflection rays should be cast for non-reflective surfaces

Metallic Reflections

- perfectly polished metal objects alter the color of the objects they reflect

$$L = L_p + k_m \mathrm{raycolor}(\mathrm{p} + s\mathrm{r}), \tag{10}$$

where k_m is the mirror color

Ideal Reflection

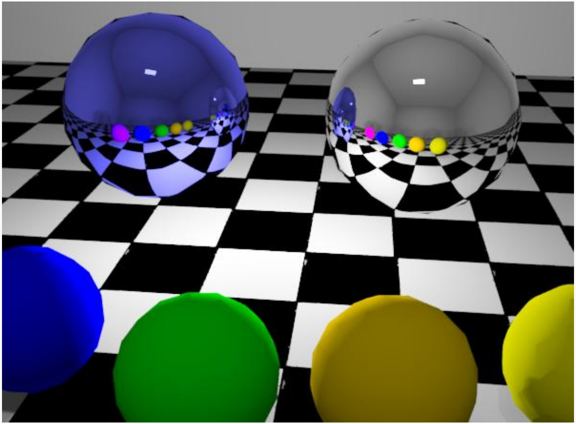


Figure 3: "Sample of metallic reflection. Model created with Google SketchUp. Rendered with IRender nXt." by AlHart released into the public domain

Absorption

- Intensity attenuation occurs if light travels through transparent media such as air, glass, long distances or through duty/fogy air
- Absorption depends on wavelength of light
- In ray tracing absorption is modeled by reducing the intensities of red, green and blue light by

$$\begin{aligned} e^{-a_r t} \\ e^{-a_g t} \\ e^{-a_b t}, \end{aligned} \tag{11}$$

  where t is the distance the ray travels through the absorbing medium and a_r, a_g, and a_b are the color specific absorption coefficients

Refraction

- Whenever light hits a dielectric surface such as glass or plastic, part of the light is reflected at the surface and a part is transmitted into the medium
- The part reflected off the surface is determined by the reflectivity R of the medium
- The part transmitted is given by the transitivity $T = 1 - R$
- both depend on the refractive indices n_1 and n_2 of the two media
    - air: $n = 1$
    - water: 1.33 to 1.34
    - window glass: 1.51
    - diamond: 2.42

Refraction

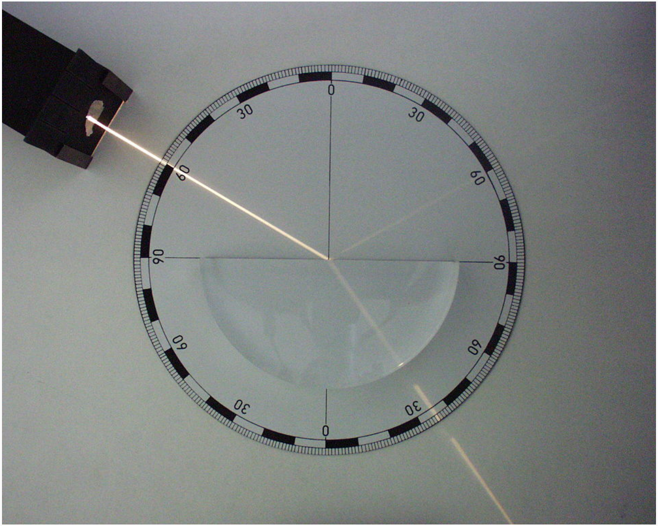


Figure 4: "Refraction" by Fizped licensed under CC BY-SA 3.0

Refraction

- The reflectivity can be approximated according to Schlick's formula by

$$R(\varphi_1) = R_0 + (1 - R_0)(1 - \cos\varphi_1)^5, \tag{12}$$

  where φ_1 is the angle, at which the ray hits the surface boundary and $R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$
- The direction of transmission is given by

$$\mathrm{t} = \frac{n_1(\mathrm{d} - \mathrm{n}(\mathrm{d} \cdot \mathrm{n}))}{n_2} - \mathrm{n}\sqrt{1 - \frac{n_1^2(1 - (\mathrm{d} \cdot \mathrm{n})^2)}{n_2^2}}, \tag{13}$$

  where n and d are the normalized surface normal and incoming ray direction
- Whenever the discriminant is negative total reflection occurs

Refraction

Figure 5: "Green sea turtle, Chelonia mydas and his total internal reflection" by Brocken Inaglory licenced under CC BY-SA 4.0.

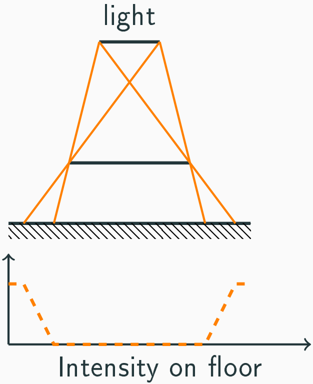
Refraction

if hit dielectric at p **then**
calculate reflection direction r
calculate color attenuation $k_r = e^{-a_r t}$
calculate color attenuation $k_g = e^{-a_r t}$
calculate color attenuation $k_b = e^{-a_r t}$
if above total reflection angle **then**
return $k\,\mathrm{raycolor}(\mathrm{p} + s\mathrm{r})$
else
calculate reflectivity R
calculate transmission direction t
return $k(R\,\mathrm{raycolor}(\mathrm{p} + s\mathrm{r}) + (1 - R)\mathrm{raycolor}(\mathrm{p} + s\mathrm{t}))$
end if
end if

Soft Shadows

- Rather than point like light sources most natural or artificial light sources expand over a certain area A
- The set of points where the light source is completely obstructed by some object are called **umbra** and the set where points are only partially obstructed are called **penumbra**
- For these light sources the question of visibility has no binary answer, occluded or not, but also partial occlusion is possible

Soft Shadows



Soft Shadows

- Let $\mathrm{f}(s,t)$, $(s,t) \in T$ be an explicit parametrization of the light surface $\mathcal{S} = \mathrm{f}(T)$ and $\mathrm{p} \in \mathbb{R}^3$ a point on the surface then
$$A_v = \int_T \mathfrak{o}(\mathrm{f}(s,t),\mathrm{p}) \left\| \frac{\partial \mathrm{f}}{\partial s} \times \frac{\partial \mathrm{f}}{\partial t} \right\|_2 dsdt, \tag{14}$$
is the surface area of the light source visible at p, where $\mathfrak{o}$ is the binary occlusion function, which is one if $\mathrm{f}(s,t)$ is visible from p and zero otherwise
- Numerical methods are used

Monte Carlo Integration
Given N uniform samples $\mathrm{x}_i \in \mathcal{S}$ on the light surface and the size of the surface area σ which would be visible from p without occlusion

$$A_v \approx \frac{\sigma}{N} \sum_{i=1}^{N} \mathfrak{o}(\mathrm{x}_i,\mathrm{p}). \tag{15}$$

Soft Shadows

- In the context of ray tracing we generate a number of rays from p to uniform distributed points of the light surface and count the fraction rays not occluded
- As the points on the light surface are chosen at random no two calculations yield the exact same result
- Number of sampling points must be chosen sufficiently high to avoid noise in the image
- Number of points must be chosen sufficiently low to avoid to large of a performance impact
- Especially for a low number of sampling points there are ways to generate the random points on the surface

Antialiasing

- The image we see in ray tracing might differ from the original, which was intended hence it is referred to as **alias**
- E.g. jagged lines, where there should be a straight one
- These unwanted effects can be reduced either on the final image in a post processing step or by building antialiasing into the ray tracer
- Instead of sampling from only one ray centered at the pixel, multiple rays can be started at random positions within the pixel and the colors averaged

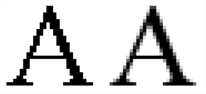


Figure 6: "Aliasing example of the "A" letter in Times New Roman" by Mwyann licensed under CC BY-SA 3.0

Sampling Points

- Sampling points on a regular grid for soft shadow calculation or anti aliasing can lead to artifacts such as **moiré patterns**
- This can be avoided by either **uniform random sampling** or **stratified sampling** (jittering)

Ambient Occlusion

- Objects which stand close next to each other will cause that parts of the ambient light are occluded for surfaces facing each other
- Ambient occlusion A models how exposed surface points are to ambient light with $A = 1$ beeing fully exposed and $A = 0$ not beeing exposed
- Let p be a point on a surface with normal vector n, Ω_p be the hemisphere centered around p then the ambient occlusion A is given by
$$A = \frac{1}{\pi} \int_{\Omega_\mathsf{p}} (\mathsf{n} \cdot \mathsf{w}) v(\mathsf{p}, \mathsf{w}) d^3 w, \tag{16}$$
where $v(\mathsf{p}, \mathsf{w})$ be the visibility function, which is zero if p is occluded in the direction w and one otherwise.
- Again possible to use Monte Carlo integration

Ambient Occlusion

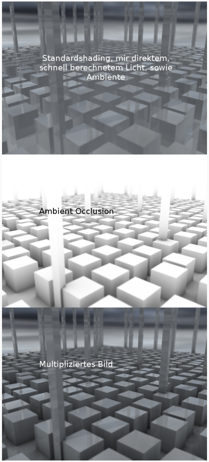


Figure 7: "Zeigt ein praktisches Anwendungsbeispiel für Ambient Occlusion" by TheWuse licensed under CC BY-SA 3.0.

Blurry Reflections

- Most surfaces are not perfectly polished but have some degree of roughness
- To simulate this a number of reflection rays can be cast, each of which is a little bit disturbed off the ideal reflection axis r
  1. Create an orthonormal basis $\{\mathrm{u}, \mathrm{v}, \mathrm{w}\}$ with $\mathrm{w} = \frac{\mathrm{r}}{\|\mathrm{r}\|_2}$
  2. Create random point inside circle with radius R in uv-plane

$$\varphi = 2\pi \mathrm{rand}() \tag{17}$$

$$r = R\sqrt{\mathrm{rand}()} \tag{18}$$

  3. calculate disturbed direction

$$\mathrm{r}' = \mathrm{r} + r\cos\varphi\mathrm{u} + r\sin\varphi\mathrm{v} \tag{19}$$

- This will blur reflected objects
- The larger the radius R the more brushed surfaces appear

Blurry Reflections

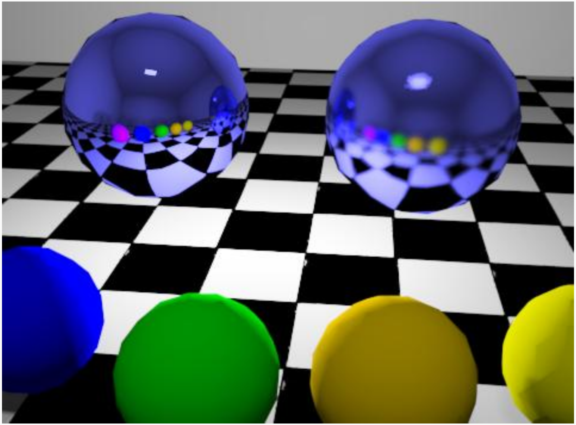


Figure 8: "Sample of blurry reflection. Model created with Google SketchUp. Rendered with IRender nXt." by AlHart released into the public domain

Motion Blur

- Motion blur is an effect that occurs when a scene changes during the capturing process
- Most prominent in photography under low light conditions (long exposure)
- It can be simulated by tracing a scene at multiple times during an time interval $[t_1, t_2]$ and averaging these images
- To avoid artifacts (e.g. spinning wheel standing still due to aligned rotation and sampling rate) it is a good idea to sample the scene at random time points

Motion Blur

Figure 9: "A London bus passes a telephone box on Haymarket, Westminster, London, UK" by E01 licensed under CC BY-SA 2.0.

Depth of Field

- Another effect observed in photography are planes, which are out of focus, which are heavily blurred
- Effect scales with the size of the aperture of an optical system
- This effect can be modeled quite simple
  - Project the plane which determines the ray direction onto a parallel plane, which should be in focus
  - Use a small area as eye position instead of a single point
  - sample multiple rays per pixel with uniform random eye positions on the eye plane and average the resulting pixel values
- To model a real camera the eye sampling area should be a sphere
- The effects will be very similar for a square lens

Depth of Field

Figure 10: "A 3D model of organic molecules created using Rhinoceros 3D and rendered with Vray. [...] The ray-tracing depth was set to 9, and caustics are enabled. The index of refraction of the glass material is set to 1.55." by Purpy Pupple lisenced under CC BY-SA 3.0.

Summary

- Shaders can be build from the parts discussed so far
- Certain properties of light are usually not considered
  - Refractive index depends on wavelength
  - Light has a property called polarization, which influences its behaviour (refraction)
  - Light is a wave and may interfere

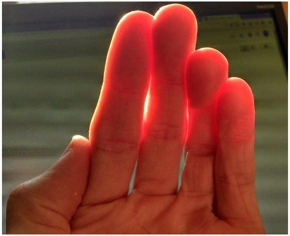


Figure 11: "Example of Subsurface Scattering of light in a real world photograph" by Davepoo2014 licensed under CC BY-SA 4.0.