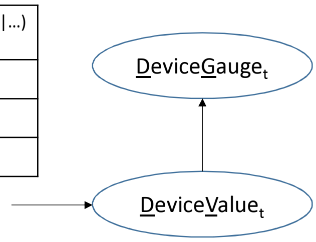
Exercise 7

1. Give a DBN example with CPTs that show how transient failure of measurement devices can be handled. Use CPTs of size 3x3 and write down an expression that shows the effect of your design

DV	P(DG=0|…)	P(DG=1|…)	P(DG=2|…)
0			
1			
2			



Assume $\mathbf{P}(DV_t|DV_{t-1})$=<0.05,0.15,0.8>

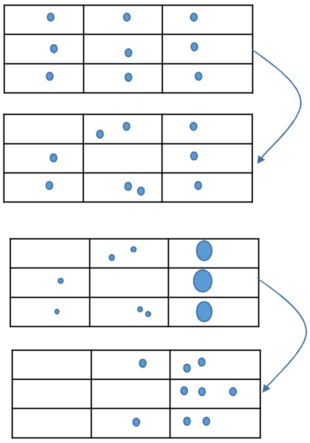
$P(DV_t|DG_t=0) = \alpha\, \mathbf{P}(DG_t|DV_t)\, \mathbf{P}(DV_t|DV_{t-1})$

$= \alpha$ <0.8,0.1,0.1> * <0.05,0.15,0.8>

≈ <0.3,0.1,0.6>

Have to find a distribution, that allows false readings.

2. Describe the main steps of particle filtering. Also, compare particle filtering with Likelihood weighting



Assume random walk $P(x_{t+1}|x_t)$.
Propagate each sample forward, based on the transition model.

Assume we observe *wall right* and nearly good sensors.
Update the weight of the samples w.r.t the observation.

Resample based on distribution of possible states and weights.

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3. What would happen if resampling is skipped?

If you just keep your old particles around forever without resampling then, what happens is that your particles drift around according to your motion model (transition probabilities for the next time step), but other than their weights, they are unaffected by your observations. Highly unlikely particles will be kept around and transitioned to more unlikely states, and you might only have say, one particle in the area of high probability of your posterior.

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4. Given the following possible gambles. What is the relationship between the gambles in terms of dominance.

Fair die, three gambles and payoffs

State (die result)	1	2	3	4	5	6
Gamble A wins $	1	1	2	2	2	2
Gamble B wins $	1	1	1	2	2	2
Gamble C wins $	3	3	3	1	1	1

Random variable **A** has stochastic dominance over random variable **B**
if for any outcome *x*, **A** gives at least as high a probability of receiving at least *x* as does **B**,
and for some *x*, **A** gives a higher probability of receiving at least *x*.

$\Pr(A \geq x) \geq \Pr(B \geq x)$ for all x, and for some x $\Pr(A \geq x) > \Pr(B \geq x)$

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Fair die, three gambles and payoffs

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$\Pr(A \geq x) \geq \Pr(B \geq x)$ for all x,
and for some x $\Pr(A \geq x) > \Pr(B \geq x)$

A dominates B? YES

$\Pr(A \geq 1) = \Pr(B \geq 1) = 1$
$\Pr(A \geq 2) = 4/6 > \Pr(B \geq 2) = 3/6$
$\Pr(A \geq 3) = \Pr(B \geq 3) = 0$

C dominates B? YES

$\Pr(C \geq 1) = \Pr(B \geq 1) = 1$
$\Pr(C \geq 2) = \Pr(B \geq 2) = 3/6$
$\Pr(C \geq 3) = 3/6 > \Pr(B \geq 3) = 0$

A dominates C? NO

$\Pr(A \geq 1) = \Pr(C \geq 1) = 1$
$\Pr(A \geq 2) = 4/6 > \Pr(C \geq 2) = 3/6$
$\Pr(A \geq 3) = 0 < \Pr(C \geq 3) = 3/6$

Gambles A and C cannot be **ordered** relative to each other on the basis of stochastic dominance

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9. Show that the judgments B $\succ$ A and C $\succ$ D in the Allais paradox violate the axiom of substitutability.

A : 80% chance of $4000
B : 100% chance of $3000

C : 20% chance of $4000
D : 25% chance of $3000

Most people would choose the sure thing B.

Most people would choose C.

- **Substitutability:** If an agent is indifferent between two lotteries, A and B, then there is a more complex lottery in which A can be substituted with B.

This also holds for $\succ$

$$(A \sim B) \Rightarrow [p : A; (1-p) : C] \sim [p : B; (1-p) : C]$$

EV[A]= 0.8*4000 = 3200

EV[B]= 1*3000

EV[C]= 0.2*4000 = 800

EV[D]= 0.25*3000 = 750

C=[25% A; 75% $0] → EV=800

D=[25% B; 75% $0] → EV=750

B $\succ$ A

C $\succ$ D

D $\succ$ C

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