Intelligent Autonomous Agents and Cognitive Robotics

Topic 10: Agent*S* and Game Theory
Topic 11: Social Choice (Preference Aggregation)

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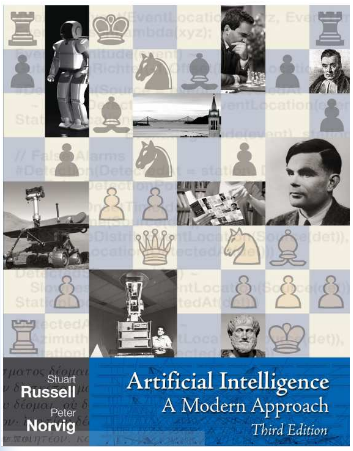
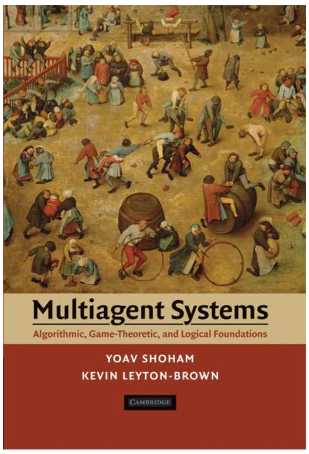
Literature

- Chapter 17



- Chapter 3



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Game Theory

- So far we looked at uncertainty of *actions* and *sensors*
- Now, uncertainty due to the **behavior** of other *agents* !!!!!

→ Game theory

Game Theory: The Basics

- **A game:** Formal representation of a situation of *strategic interdependence* (extension of Adversarial search)
  - Set of agents, I (|I|=n)
    - AKA players
  - Each agent, j, has a set of actions, Aj
    - AKA moves
  - Actions define outcomes
    - For each possible action there is an outcome → state.
  - Outcomes define payoffs
    - Agents' derive utility from different outcomes. Utilities can be the same or different.

Normal form game*
(matching pennies)

Agent 2

Action

Outcome

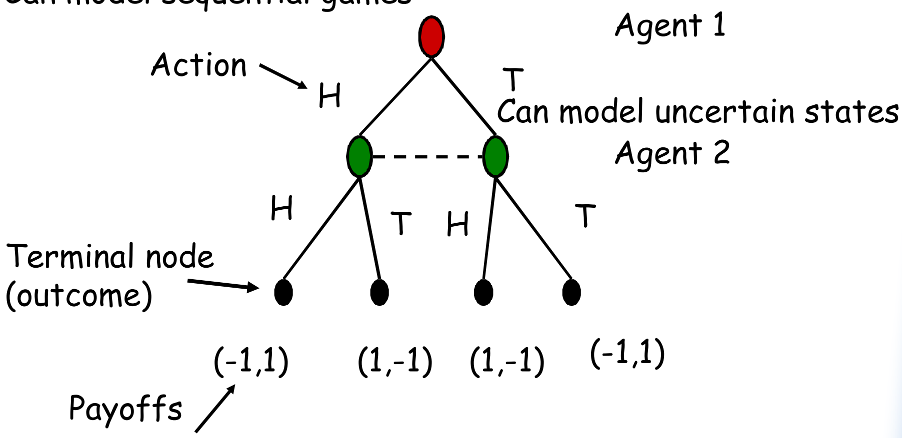
Payoffs

		H	T
Agent 1	H	-1, 1	1, -1
	T	1, -1	-1, 1

*aka strategic form, matrix form

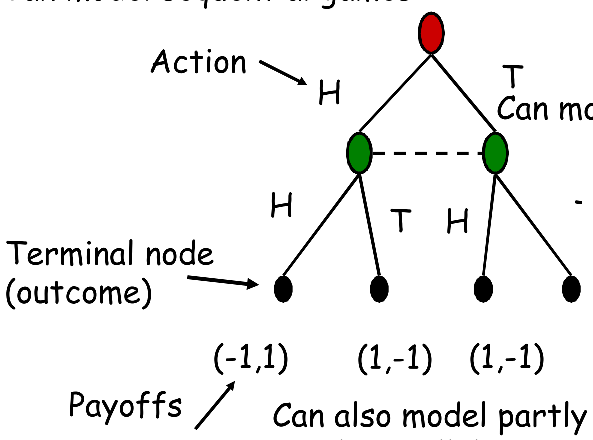
Extensive form game
(matching pennies)

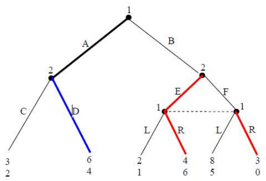
Can model sequential games



Extensive form game
(matching pennies)

Can model sequential games





Can also model partly sequential & partly parallel situations

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Strategies (aka Policies)

- Strategy:
  - A strategy, s_j, is a **complete contingency plan** (policy); defines actions agent **j** should take for all possible states of the world
- Strategy profile:
  - $\mathbf{s = (s_1,\ldots,s_n)}$ (all agents)
  - $\mathbf{s_{-i} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n)}$ (all agents without **i**)
- Utility function: $u_i(\mathbf{s})$
  - Note that the utility of an agent depends on the strategy profile, not just its own strategy
  - We assume agents are **expected utility maximizers**

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Normal form game*

(matching pennies)

Strategy for agent 2: T

Strategy for agent 1: H

Agent 1 \ Agent 2	H	T
H	-1, 1	1, -1
T	1, -1	-1, 1

Strategy profile (H,T)

U1((H,T))=1
U2((H,T))=-1

*aka strategic form, matrix form

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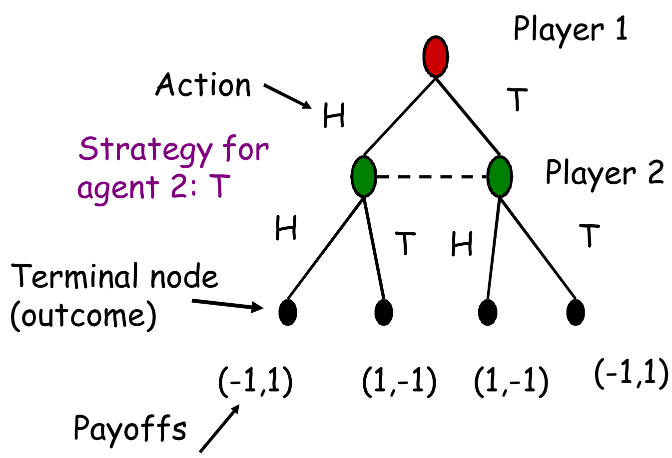
Extensive form game

(matching pennies)

Strategy for agent 1: T

Strategy for agent 2: T



Strategy profile: (T,T)

U1((T,T))=-1
U2((T,T))=1

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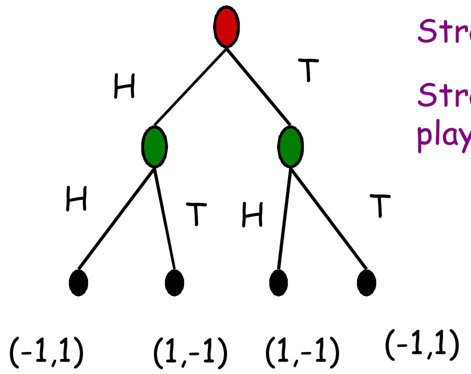
Extensive form game
(matching pennies, seq moves)

Recall: A strategy is a contingency plan for all states of the game



Strategy for agent 1: T

Strategy for agent 2: H if 1 plays H, T if 1 plays T (H,T)

Strategy profile: (T,(H,T))

U1((T,(H,T)))=-1

U2((T,(H,T)))=1

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Dominant Strategies

- Recall that
  - Agents' utilities depend on what strategies other agents are playing
  - Agents' are expected utility maximizers
- Agents' will play best-response strategies for s_{-i}

s_i^* is a best response if $u_i(s_i^*,s_{-i}) \geq u_i(s_i',s_{-i})$ for all s_i'

- A dominant strategy is a best-response for all s_{-i}
  - They do not always exist
  - Inferior strategies are called dominated

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Dominant Strategy Equilibrium

- A *dominant strategy equilibrium* is a strategy profile where the strategy for each player is dominant
  - $s^*=(s_1^*,\ldots,s_n^*)$
  - $u_i(s_i^*,s_{-i})\geq u_i(s_i',s_{-i})$ for all i, for all s_i', for all s_{-i}
- **GOOD**:
  Agents do not need to counterspeculate!

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Example: Prisoner's Dilemma

- Two people are arrested for a crime. A prosecutor offers each a deal: if you **testify** against your partner as the leader of a burglary ring, you'll go free for being the cooperative one, while your partner will serve 10 years in prison. However, if both **testify** against each other, they both get 5 years. If both **refuse**, each get 1 year.

	A: testify	A: refuse
B:testify	B = -5 A = -5	B = 0 A = -10
B:refuse	B = -10 A = 0	B = -1 A = -1

Dom. Str. Eq

Pareto Optimal Outcome

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Example: Bach or Stravinsky

aka Battle of sexes

- A couple likes going to two concerts. One loves Bach but not Stravinsky. The other loves Stravinsky but not Bach. However, they prefer being together than being apart.

	B	S
B	2,1	0,0
S	0,0	1,2

No dom. str. equil.

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Nash Equilibrium

- Sometimes an agent's best-response depends on the strategies other agents are playing
  - No dominant strategy equilibria
- A strategy profile is a **Nash equilibrium** if no player has incentive to deviate from his strategy given that others do not deviate:
  - for every agent i, $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i}) \rightarrow s_i^*$ is a best response to s_{-i}

	B	S
B	2,1	0,0
S	0,0	1,2

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How to find (Nash) Equilibria

- Can agents rule out strategies?
  - Strategies an agent will not play
- Get rid of those strategies
  - Maybe there will exist a single solution

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Example

	r	l	c
U	3,-3	7,-7	9,-15
D	9,-9	8,-8	10,-10

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Iterated Elimination of Dominated Strategies

- Let Ri⊆Si be the set of removed strategies for agent i
- Initially Ri=Ø
- Choose agent i, and strategy s_i such that $s_i \in S_i \backslash R_i$ and there exists $s_i' \in S_i \backslash R_i$ such that
  $u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i} \backslash R_{-i}$
- Add s_i to R_i, continue

- **Thm:** If a unique strategy profile, s*, survives then it is a Nash Eq.
- **Thm:** If a profile, s*, is a Nash Equilibrium then it must survive iterated elimination.

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Nash Equilibrium

- Criticisms
  - They may not be unique (Bach or Stravinsky)
    - Ways of overcoming this
      - Refinements of equilibrium concept, Mediation, Learning
  - Do not exist in all games (in form defined)
  - They may be hard to find
  - People don't always behave based on what equilibria would predict (ultimatum games and notions of fairness,…)

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Example: Matching Pennies

	H	T
H	-1, 1	1,-1
T	1,-1	-1, 1

There is NO (Nash) strategy in pure strategies

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Example: Bach Stravinsky

	B	S
B	2,1	0,0
S	0,0	1,2

If I do not know, what the other agent is doing, and
if communication is not possible, what should the agents do

So far we have talked only about **pure** strategy equilibria.

Not all games have pure strategy equilibria. Some equilibria are **mixed** strategy equilibria.

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Example: Bach Stravinski

			Husband q B	1-q S
	p	B	2, 1	0, 0
Wife	1-p	S	0, 0	1, 2

Mixed strategies can help if no communication is possible. Want to play a strategy so that the other is indifferent playing a pure strategy (B or S).

EU_{HB} = 1p + 0 (1-p)
EU_{HS} = 0p+ 2 (1-p)

$EU_{HB} = EU_{HS}$

p = 2-2p
p=2/3 (wife has mixed <2/3;1/3>)

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Example: Bach Stravinski

			Husband q B	1-q S
	p	B	2, 1	0, 0
Wife	1-p	S	0, 0	1, 2

Mixed strategies can help if no communication is possible. Want to play a strategy so that the other is indifferent playing a pure strategy (B or S).

EU_{WB} = 2q + 0 (1-q)
EU_{WS} = 0q + 1(1-q)

$EU_{WB} = EU_{WS}$
2q = 1-1q
q=1/3 (husband has mixed <1/3;2/3>)

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Example: Bach Stravinski

- If Husband **strictly** plays B with q=1/3
  - Which distribution can his wife play
  - $EU_w(p,1\text{-}p)$ = *1/3*p*2 + 2/3 *p * 0 + 1/3*(1-p)*0 + 2/3*(1-p)*1 =*
    *2/3*p + 2/3 -2/3p =*
    2/3
    any distribution leads to 2/3 in average

		1/3 B	2/3 S
p	B	2, 1	0, 0
1-p	S	0, 0	1, 2

Example: Bach Stravinski

			Husband	
			q B	1-q S
	p	B	2, 1	0, 0
Wife	1-p	S	0, 0	1, 2

$EU_{WB} = 2q + 0\,(1-q)$
$EU_{WS} = 0q + 1(1-q)$

$EU_{WB} = EU_{WS}$
$2q = 1-1q$
$q=1/3$

husband has mixed strategy <1/3;2/3>
wife has mixed strategy <2/3;1/3>

Example: Bach Stravinski

- If Husband **strictly** plays B with q=1/3
  - Which distribution should wife play
  - $Eu_w(p,1-p) = 2/3$
- If Husband deviates q<1/3
  - Wife deviates plays **S**
- If Husband q>1/3
  - Wife plays **B**
- Equilibrium: {(2/3,1/3);(1/3,2/3)}

		Husband q B	Husband 1-q S
Wife	p B	2, 1	0, 0
Wife	1-p S	0, 0	1, 2

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Mixed strategy equilibria

- Mixed strategy:
  $\sigma_i \in \Sigma_i$ defines a probability distribution over S_i
- Strategy profile: $\sigma=(\sigma_1,\ldots,\sigma_n)$
- Expected utility: $u_i(\sigma)=\sum_{s\in S}(\Pi_j\ \sigma(s_j))u_i(s)$
- Nash Equilibrium:
  - σ^* is a mixed Nash equilibrium if

  $u_i(\sigma^*_i, \sigma^*_{-i}) \geq u_i(\sigma_i, \sigma^*_{-i})$ for all $\sigma_i \in \Sigma_i$, for all i

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Mixed Nash Equilibrium

- Thm (Nash 50):
  - Every game in which the strategy sets $S_1,\ldots,S_n$ have a finite number of elements, has a **mixed strategy equilibrium**.
- Finding Nash Equilibria is another problem
  - "Together with prime factoring, the complexity of finding a Nash Eq is the most important concrete open question …" (Papadimitriou)

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Bayesian-Nash Equil

(Harsanyi 68)

- So far we have assumed that agents have complete information about each other (including payoffs)
  - Very strong assumption!
- Assume agent i has type $\theta_i \in \Theta_i$, defines the payoff $u_i(s, \theta_i)$
- Agents have common prior over distribution of types $p(\theta)$
  - Conditional probability $p(\theta_{-i} | \theta_i)$ (obtained by Bayes Rule when possible)

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Battle of the sexes

- Shopping or Basketball?
- Sally knows Kevins type
  Kevin does not know Sally's type but possible types.

What should Sally play?

Sally a basketball fan

Sally \ Kevin	Basketball	Shopping
Basketball	3, 2	2,1
Shopping	0,0	1, 3

Her **dominant** strategy!

Sally a shopping fan

Sally \ Kevin	Basketball	Shopping
Basketball	1, 2	0,1
Shopping	2,0	3, 3

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Battle of the sexes

- Sally should play her **dominant** strategy

$\Theta_1 = \{\theta_{11,}, \theta_{12}\}$, $\Theta_2 = \{\theta_2\}$,

$P(\theta_{11},\theta_2) = p$ $\quad P(\theta_{12},\theta_2) = (1-p)$

2p +0(1-p) > 1p + 3 (1-p) basketball vs shopping

2p > -2p+3

p > 3/4

Sally \ Kevin	Basketball	Shopping
Basketball	3, $\boxed{2}$	2 $\boxed{1}$
Shopping	0,0	1, 3

Sally \ Kevin	Basketball	Shopping
Basketball	1, 2	0,1
Shopping	2 $\boxed{0}$	3, $\boxed{3}$

If p>3/4 Basketball
If p<3/4 Shopping
If p=3/4 ??

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Battle of the sexes

- Sally's decision depends on her known type
- Kevin's decision depends on p

$$S_2^*(\theta_2) = \begin{cases} \text{Basketball} & \text{if } p>3/4 \\ (q,\ 1-q), q \in [0,1] & \text{if } p=3/4 \\ \text{Shopping} & \text{if } p<3/4 \end{cases}$$

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Bayesian-Nash Equil

- Strategy: $\sigma_i(\theta_i)$ is the (mixed) strategy agent i plays if its type is θ_i .
- Strategy profile: $\sigma=(\sigma_1,\ldots, \sigma_n)$
- Expected utility:
  - $U_i(\sigma_i(\theta_i),\sigma_{-i}(),\ \theta_i)=\Sigma_{\theta_{-i}}\ p(\theta_{-i}|\theta_i)u_i(\sigma_i(\theta_i),\sigma_{-i}(\theta_{-i}),\theta_i)$
- Bayesian Nash Eq: Strategy profile σ^* is a Bayesian-Nash Equilibrium if for all i, for all θ_i,
  $U_i(\sigma^*_i(\theta_i),\sigma^*_{-i}(),\theta_i) \geq U_i(\sigma_i(\theta_i),\sigma^*_{-i}(),\theta_i)$

  (best responding w.r.t. its beliefs about the types of the other agents, assuming they are also playing a best response)

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Last time

- Definition of games
- Strategies & Strategy profiles
  - Dominant strategy equilibrium

    $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall\, s_i',\ \forall\, s_{-i},\ \forall\, i,$
  - Nash equilibrium

    $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall\, s_i',\ \forall\, i,$
  - Mixed Nash strategy equilibrium

    $u_i(\sigma_i^*, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})\ \forall\, \sigma_i',\ \forall\, i,$
  - Bayesian Nash equilibrium

    $u_i(\sigma^*_i(\theta_i), \sigma^*_{-i}(), \theta_i) \geq u_i(\sigma_i(\theta_i), \sigma^*_{-i}(), \theta_i)$

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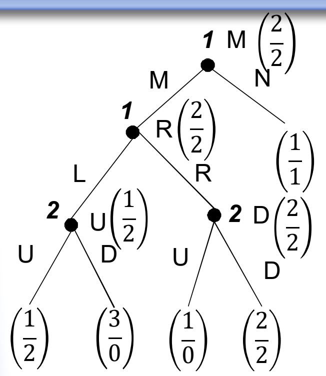
Extensive Form Games



Any finite game of perfect information has a pure strategy Nash equilibrium.

It can be found by backward induction.

How to find a Nash Equilibrium?
By backward induction!
Have to define an action for every choice point.
(MR, UD)

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Subgame perfect equilibrium & credible threats

- Proper subgame = subtree (of the game tree) whose root is alone in its information set (agent knows his state)
- Subgame perfect equilibrium
  - Strategy profile that is in Nash equilibrium in every proper subgame (including the root), whether or not that subgame is reached along the equilibrium path of play

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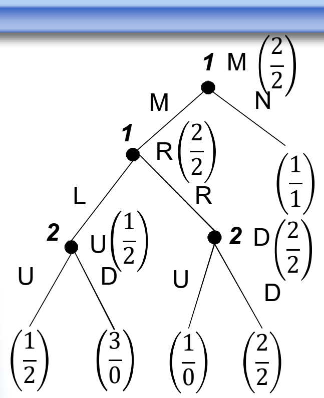
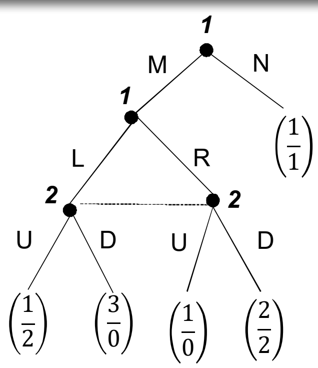
Subgame perfect equilibrium



(MR, UD)



What is the strategy now?

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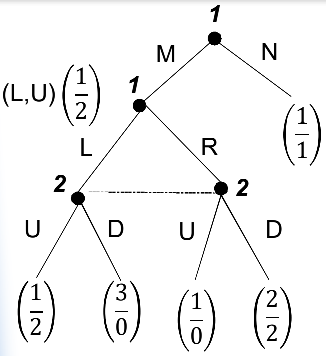
Subgame perfect NE equilibrium



(MR, UD)

What is the strategy now?

	U	D
L	1,2	3,0
R	1,0	2,2

If U best is L or R
If D best is L
If L best is U
If R best is D
(L,U) is the NE

(ML, U) and (NL, U) are the SPNE of the game
$\binom{1}{2}$ $\binom{1}{1}$

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Non creditable threats

- A firm is deciding whether to enter the market, which another firm currently has a monopoly over.
- If the firm enters, the monopolist chooses whether to accept it or declare a price war.
  - The firm only wants to enter if the monopolist won't engage in a price war
  - A price war is unprofitable for the monopolist

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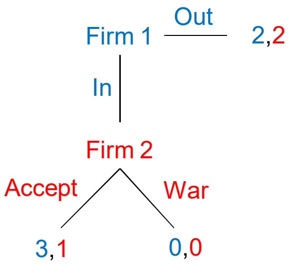
Non creditable threats



	Accept	War
In	3,1	0,0
Out	2,2	2,2

Firm 2 announce to make a price war if Firm 1 enters.
(out, war) is a Nash equilibria.

But, it is not subgame perfect →
This is a non creditable thread

Social choice theory

- Study of decision problems in which a group has to make the decision
- The decision affects all members of the group
  - Their opinions! should count
- Applications:
  - Political elections
  - Note that outcomes can be vectors
    - Allocation of money among agents, allocation of goods, tasks, resources…
- CS applications:
  - Multiagent planning [Ephrati&Rosenschein]
  - Accepting a joint project, rating Web articles [Avery,Resnick&Zeckhauser]
  - …

Criteria for evaluating multiagent systems

- Social welfare: $\max_{outcome} \sum_i u_i(outcome)$
- Surplus: social welfare of outcome – social welfare of status quo
  - Zero sum games have 0 surplus. Markets are not zero sum
- Pareto efficiency: An outcome o is Pareto efficient if there exists no other outcome o' s.t. some agent has higher utility in o' than in o and no agent has lower
  - Implied by social welfare maximization
- Individual rationality: Participating in the negotiation (or individual deal) is no worse than not participating
- Stability: No agents can increase their utility by changing their strategies
- Symmetry: No agent should be inherently preferred, e.g. dictator

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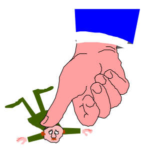
Assumptions

1. Agents have preferences over alternatives
   - Agents can rank order the outcomes
     - a>b>c=d is read as "a is preferred to b which is preferred to c which is equivalent to d"
2. Voters are sincere
   - They truthfully tell the center their preferences
3. Outcome is enforced on all agents



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Voting

- Majority decision:
  - If more agents prefer a to b, then a should be chosen
- Two outcome setting is easy
  - Choose outcome with more votes!
- What happens if you have 3 or more possible outcomes?

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Case 1: Agents specify their top preference

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Election System

- Plurality Voting
  - One name is ticked on a ballot
  - One round of voting
  - One candidate is chosen

Is this a "good" system?

What do we mean by good?

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Example: Plurality (Canada)

- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result: Lib 10, NDP 6, C 5
  - But a majority of voters (11) prefer all other parties more than the Libs!

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What can we do?

- Majority system
  - Works well when there are 2 alternatives
  - Not great when there are more than 2 choices
- Proposal:
  - Organize a series of votes between 2 alternatives at a time
  - How this is organized is called an Agenda
    - Or a cup (often in sports)

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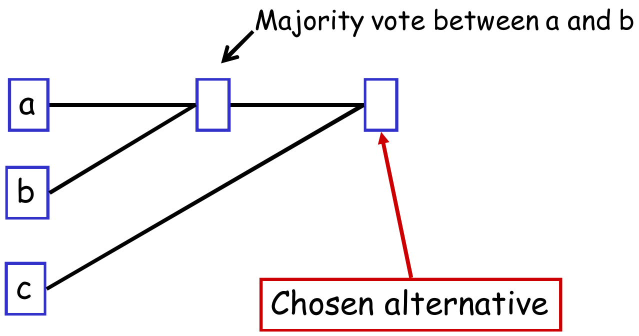
Agendas

- 3 alternatives {a,b,c}
- Agenda a,b,c



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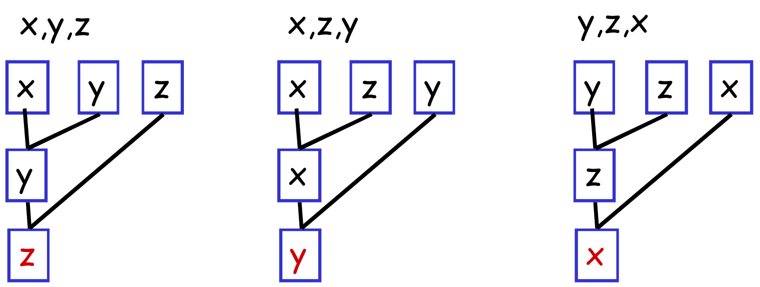
Example: Agenda

- *Binary protocol (majority rule)* Three types of agents:
  1. x > z > y (35%)
  2. y > x > z (33%)
  3. z > y > x (32%)

Chairman defines order



- Power of agenda setter (e.g. chairman)

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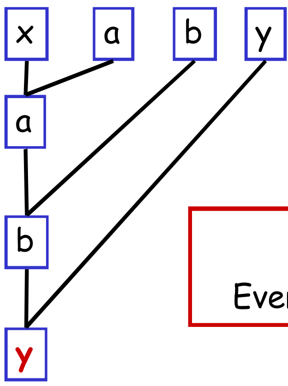
Pareto dominated winner paradox

Agents:
1. x > y > b > a
2. a > x > y > b
3. b > a > x > y



BUT

Everyone prefers x to y!

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Case 2: Agents specify their complete preferences

Maybe the problem was with the ballots!

Ballot

X>Y>Z

Now have more information

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Condorcet

Marie Jean Antoine Nicolas Caritat, Marquis de Condorcet

- Proposed the following
  - Compare each pair of alternatives
  - Declare "a" is socially preferred to "b" if more voters strictly prefer a to b
- Condorcet Principle: If one alternative is preferred to all other candidates then it should be selected

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Example: Condorcet

- 3 candidates
  - Lib, NDP, C
- 21 voters with the preferences
  - 10 Lib>NDP>C
  - 6 NDP>C>Lib
  - 5 C>NDP>Lib
- Result:
  - NDP win! (11/21 prefer them to Lib, 16/21 prefer them to C)

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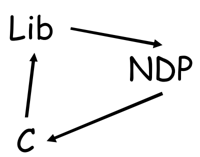
A Problem

- 3 candidates
  - Lib, NDP, C
- 3 voters with the preferences
  - Lib>NDP>C
  - NDP>C>Lib
  - C>Lib>NDP
- Result:
  - No Condorcet Winner



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Borda Count

- Each ballot is a list of ordered alternatives
- Over all ballots compute the rank of each alternative
- Rank order alternatives based on decreasing sum of their ranks

A>B>C	A: 4
A>C>B	B: 8
C>A>B	C: 6

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Borda Count

- Simple. Only counting ranks
- Always a Borda Winner, but have to define a solution for ties.
- BUT does not always choose Condorcet winner!
- 3 voters
  - 2: b>a>c>d
  - 1: a>c>d>b

Borda scores:

a:5, b:6, c:8, d:11

Therefore a wins

BUT b is the Condorcet winner

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Another example

- Borda rule with 4 alternatives
- Agents:
  1. x > c > b > a
  2. a > x > c > b
  3. b > a > x > c
  4. x > c > b > a
  5. a > x > c > b
  6. b > a > x > c
  7. x > c > b > a

- x=13
- a=18
- b=19
- c=20

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The winner is dropped

- X went out → Remove x:

1. x > c > b > a	1. c > b > a
2. a > x > c > b	2. a > c > b
3. b > a > x > c	3. b > a > c
4. x > c > b > a	4. c > b > a
5. a > x > c > b	5. a > c > b
6. b > a > x > c	6. b > a > c
7. x > c > b > a	7. c > b > a

- x=13, a=18, b=19, c=20
- c=13
- b=14
- a=15

Inverted order paradox

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Borda rule vulnerable to irrelevant alternatives

- Three types of agents:

x	y
35	70
66	33
64	32
165	135
second	first

1. x > y (35%)
2. y > x (33%)
3. y > x (32%)

- Borda winner is y

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Borda rule vulnerable to irrelevant alternatives

- Three types of agents:

x	y
35	70
66	33
64	32
165	135
second	first

1. x > z > y (35%)
2. y > x > z (33%)
3. z > y > x (32%)

x	y	z
35	105	70
66	33	99
96	64	32
197	202	201
first	third	second

- Borda winner is y
- Add z Borda winner is x

The social preferences between alternatives x and y depend only on the individual preferences between x and y

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Desirable properties for a voting protocol

- No dictators
- Universality (unrestricted domain)
  - It should work with any set of preferences
- Independence of irrelevant alternatives
  - The comparison of two alternatives should depend only on their standings among agents' preferences, not on the ranking of other alternatives
- Pareto efficient
  - If all agents prefer x to y then in the outcome x should be preferred to y

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Arrow's Theorem (1951)

- **Thrm.** If there are 3 or more alternatives and a finite number of agents then there is **no** protocol which satisfies the 4 desired properties
- **Thrm**. Let $|O| \geq 3$, any social welfare function W that is Pareto efficient and independent of irrelevant alternatives is dictatorial.

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Take-home Message

- Despair?
  - No ideal voting method
  - That would be boring!
- A group is more complex than an individual
- Weigh the pro's and con's of each system and understand the setting they will be used in
- Do not believe anyone who says they have the best voting system out there!