Exercise 8

1. Consider the following three-state, four-decision problem with the following payoff table

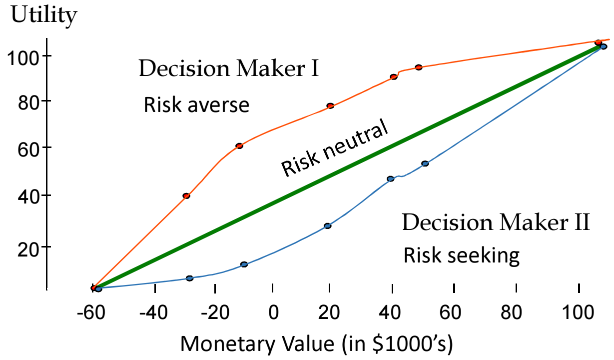
	S1	S2	S3
D1	+100 000	+40 000	-60 000
D2	+50 000	+20 000	-30 000
D3	+20 000	+20 000	-10 000
D4	+40 000	+20 000	-60 000

the probabilities for the three states are
P(S1) =0.5 P(S2)=0.3 P(S3)=0.2
Suppose two decision makers have the following utility values:

	Utility	
Amount	Decision Maker I	Decision Maker II
\$100,000	100	100
\$50,000	94	58
\$40,000	90	50
\$20,000	80	35
-\$10,000	60	18
-\$30,000	40	10
-\$60,000	0	0

- Draw the utility curves

Amount	Decision Maker I	Decision Maker II
\$100,000	100	100
\$50,000	94	58
\$40,000	90	50
\$20,000	80	35
-\$10,000	60	18
-\$30,000	40	10
-\$60,000	0	0



- Expected Utility: Decision Maker I

$$EU(D_1) = .5(100) + .3(90) + .2(\ 0) = 77.0$$
$$EU(D_2) = .5(\ 94) + .3(80) + .2(40) = 79.0$$
$$EU(D_3) = .5(\ 80) + .3(80) + .2(60) = 76.0$$

Decision Maker I's optimal decision is D_2.

Amount	Decision Maker I	Decision Maker II
\$100,000	100	100
\$50,000	94	58
\$40,000	90	50
\$20,000	80	35
-\$10,000	60	18
-\$30,000	40	10
-\$60,000	0	0

	s_1	s_2	s_3
d_1	+100,000	+40,000	-60,000
d_2	+50,000	+20,000	-30,000
d_3	+20,000	+20,000	-10,000
d_4	+40,000	+20,000	-60,000

P(S1) =0.5 P(S2)=0.3 P(S3)=0.2

- Expected Utility: Decision Maker II

$$EU(D_1) = .5(100) + .3(50) + .2(\ 0) = 65.0$$
$$EU(D_2) = .5(\ 58) + .3(35) + .2(10) = 41.5$$
$$EU(D_3) = .5(\ 35) + .3(35) + .2(18) = 31.6$$

Decision Maker II's optimal decision is D_1.

Amount	Decision Maker I	Decision Maker II
$100,000	100	100
$50,000	94	58
$40,000	90	50
$20,000	80	35
-$10,000	60	18
-$30,000	40	10
-$60,000	0	0

	s_1	s_2	s_3
d_1	+100,000	+40,000	-60,000
d_2	+50,000	+20,000	-30,000
d_3	+20,000	+20,000	-10,000
d_4	+40,000	+20,000	-60,000

P(S1) =0.5 P(S2)=0.3 P(S3)=0.2

- Value of the Decision Problem: Decision Maker I
  - Decision Maker I's optimal expected utility is 79 foe D2
  - He assigned a utility of 80 to +$20,000, and a utility of 60 to -$10,000.
  - Linearly interpolating in this range 1 point is worth $30,000/20 = $1,500.
  - Thus a utility of 79 is worth about $20,000 - $1,500 = $18,500.

Amount	Decision Maker I	Decision Maker II
$100,000	100	100
$50,000	94	58
$40,000	90	50
$20,000	80	35
-$10,000	60	18
-$30,000	40	10
-$60,000	0	0

- Value of the Decision Problem: Decision Maker II
  - Decision Maker II's optimal expected utility is 65 for D1.
  - He assigned a utility of 100 to $100,000, and a utility of 58 to $50,000.
  - In this range, 1 point is worth $50,000/42 = $1190.
  - Thus a utility of 65 is worth about $50,000 + 7($1190) = $58,330.

The decision problem is worth more to Decision Maker II since $58,330 > $18,500).

Amount	Decision Maker I	Decision Maker II
$100,000	100	100
$50,000	94	58
$40,000	90	50
$20,000	80	35
-$10,000	60	18
-$30,000	40	10
-$60,000	0	0

2. How can we implement the utility function in a multi-attribute problem?

In a multi-attribute setting we have to find out whether the attributes are mutual preferencial independent.

<x1,x2, x3> ≻ <x1',x2',x3> independent of the value of x3
If this holds for every pair of attributes→ mutual preferencial independent

$$\Rightarrow \quad V(x1, x2, x3, \ldots, \mathrm{xn}) = \sum_i V_i(xi)$$

If there are interdependencies between attributes, then additive utility functions often do not suffice. Multi-linear expressions:

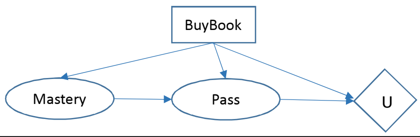
$$u(x,y)=k_1u(x)+k_2u(y)+k_1k_2u(x)u(y)$$

3. Consider a student who has the choice to buy or not buy a textbook for a course. We model this as a decision problem with one Boolean decision node, **B,** indicating whether the agent chooses to buy the book, and two Boolean nodes, **M**, indicating whether the student has mastered the material in the book, and **P**, indicating whether the student passes the course. Of course, there is also a utility node, **U**. A certain student, Sam, has an additive utility function: 0 for not buying the book and -\$100 for buying it; and \$2000 for passing the course and 0 for not passing. Sam's conditional probability estimates are as follows:

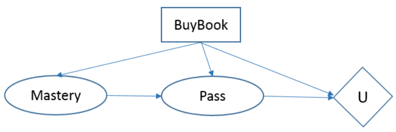
   $P(p|b,m) = 0.9$ $\quad P(m|b)=0.9$
   $P(p|b,\neg m)=0.5$ $\quad P(m|\neg b)=0.7$
   $P(p|\neg b,m)=0.8$
   $P(p|\neg b,\neg m)=0.3$

   You might think that **P** would be independent of **B** given **M.** But this course has an open-book final—so having the book helps.

   a. Draw the decision network for this problem.



$P(p|b,m) = 0.9$ $\quad P(m|b)=0.9$
$P(p|b,\neg m)=0.5$ $\quad P(m|\neg b)=0.7$
$P(p|\neg b,m)=0.8$
$P(p|\neg b,\neg m)=0.3$



b. Compute the expected utility of buying the book and of not buying it.
For **b**uying and not **b**uying we compute to **p**ass and thus not to **p**ass, by marginalization out **m**astery:

$$P(p|b) = \sum_m P(p|b,m)P(m|b) = 0.9 * 0.9 + 0.5 * 0.1 = 0.86$$

$$P(p|\neg b) = \sum_m P(p|\neg b,m)P(m|\neg b) = 0.8 * 0.7 + 0.3 * 0.3 = 0.65$$

The expected utilities are:

Sam, has an additive utility function: 0 for not buying the book and -\$100 for buying it; and \$2000 for passing the course and 0 for not passing

$$EU[b] = \sum_p P(p|b)U(p,b) = 0.86 * (2000 - 100) + 0.14 * (-100) = 1620$$

$$EU[\neg b] = \sum_p P(p|\neg b)U(p,\neg b) = 0.65 * (2000) + 0.35 * (0) = 1300$$

c. What should Sam do?

4. In the lecture we developed how to determine the EU of a decision having no additional information and how the determine the EU by adding more information links. This leads to the situation where an agent has to find out whether to buy or not buy new information. How can the value of information be computed in terms of MEUs?

What is the EU of an action and the MEU given some evidences E?

$$EU(a|E) = \sum_i U(S_i)P(S_i|E,a) \qquad MEU(a|E) = max_a \sum_i U(S_i)P(S_i|E,a)$$

What is the EU of an action and the MEU given some new evidence E_j?

$$EU(a|E, E_j = e_{jk}) = \sum_i U(S_i)P(S_i|E, E_j = e_{jk}, a)$$

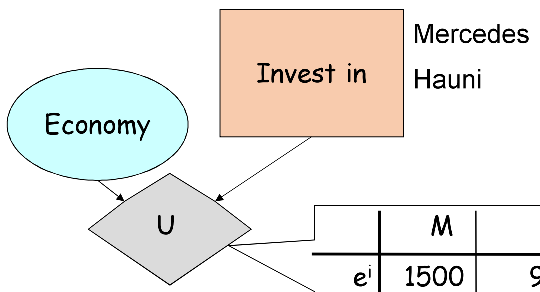
$$MEU(a|E, E_j = e_{jk}) = max_a \sum_i U(S_i)P(S_i|E, E_j = e_{jk}, a)$$

What is VPI given I can get the new evidence E_j?

$$VPI_E(E_j) = \left(\sum_k P(E_j = e_{jk}|E)\, MEU\left(a_{e_{jk}}\middle|E, E_j = e_{jk}\right)\right) - MEU(a|E)$$

5. Suppose an investment agent wants to make an investment into only one of two companies: Mercedes, Hauni. Further suppose that the economy has a 50% chance of **i**ncreasing, a 30% chance of **s**taying even, and a 20% chance of **d**ecreasing. If the economy increases the Mercedes investment will earn $1500 and Hauni will earn $900. If the economy stays even the Mercedes investment will earn $300 and the Hauni investment will earn $600. If the economy decreases the Mercedes investment will lose $800 and Hauni will lose $200.
   a. Draw a Decision network.

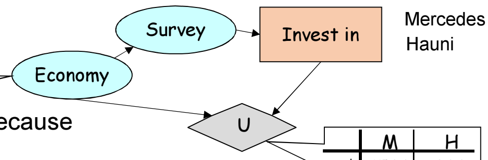
e^i	e^s	e^d
0.5	0.3	0.2



	M	H
e^i	1500	900
e^s	300	600
e^d	-800	-200

b. Compute how much the agent is willing to pay for the information of the market behavior.

e^i	e^s	e^d
0.5	0.3	0.2



	M	H
e^i	1500	900
e^s	300	600
e^d	-800	-200

We assume the survey is correct, because no information is given.
First we need the MEU without the information of the state of the market.

EU(Mercedes)= 0,5*1500+0,3*300 +0,2 *-800 = 680

EU(Hauni) = 0,5*900+0,3*600+0,2*-200 = 590

Without the survey information the MEU is 680 when choosing Mercedes

MEU_{survey} = 0,5*1500 + 0,3*600 + 0,2*-200 = 890

VPI = 890-680 = 210

	s^i	s^s	s^d
e^i	1	0	0
e^s	0	1	0
e^d	0	0	1