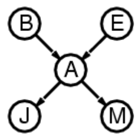
# Exercise 4
# Bayesian Networks

1. What are the basic components of a Bayesian network and what are the gains of using a BN?

- a set of nodes, one per variable
- a directed, acyclic graph (link ≈ "directly influences")
- a conditional distribution for each node given its parents:

  **P** ($X_i$ | Parents ($X_i$))
- Gain
  We can explicitly model conditional independence and independence




- The number of independent values reduces (in average) from exponential in N (number of variables) to linear in N. This also reduces the time complexity of decision making.

2

2. Suppose in a hospital an agent monitoring system keeps track of the blood sugar concentration of patients and alarms the staff if the blood sugar gets too low. The blood sugar gauge measures the blood sugar concentration of a patient.
   a. Derive and draw a Bayesian network for the problem described above and integrate failure model for devices that may fail.

   b. Suppose there are just two possible actual and measured blood sugar concentrations, **low** and **normal**. The probability that the gauge gives the correct blood sugar concentration is 0.89 when it is working, but 0.05 when it is faulty. Give the conditional probability table associated with the gauge.

   c. Suppose the alarm and blood sugar gauge are working and the alarm sounds. Derive an expression for the probability that the blood sugar is too low, in terms of the various conditional probabilities in the network.

3

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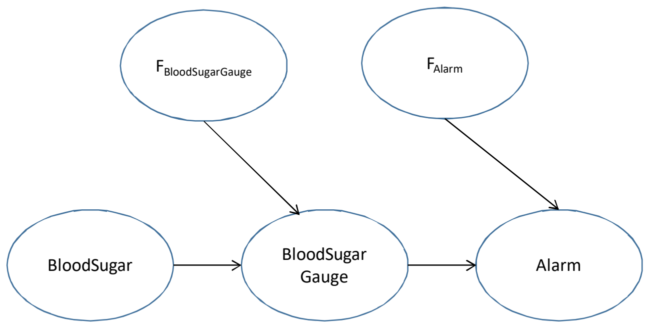
   a. Derive and draw a Bayesian network for the problem described above and integrate a failure model for devices that may fail.




4

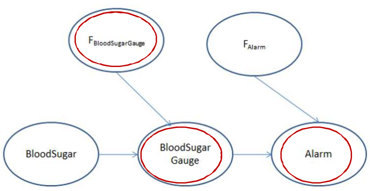
2. Suppose in a hospital an agent monitoring system keeps track of the blood sugar concentration of patients and alarms the staff if the blood sugar gets too low. The blood sugar gauge measures the blood sugar concentration of a patient.
   b. Suppose there are just two possible actual and measured blood sugar concentrations, Low and Normal. The probability that the gauge gives the correct blood sugar concentration is 0.89 when it is working, but 0.05 when it is faulty. Give the conditional probability table associated with the gauge.

| BloodSugar | $F_{BloodSugar}$ | P(BloodSugarGauge=Normal) | P(BloodSugarGauge=Low) |
|---|---|---|---|
| Normal | False | 0.89 | 0.11 |
| Normal | True | 0.05 | 0.95 |
| Low | False | 0.11 | 0.89 |
| Low | True | 0.95 | 0.05 |



2. Suppose in a hospital an agent monitoring system keeps track of the blood sugar concentration of patients and alarms the staff if the blood sugar gets too low. The blood sugar gauge measures the blood sugar concentration of a patient.
   c. Suppose the alarm and blood sugar gauge are working and the alarm sounds. Derive an expression for the probability that the blood sugar is too low, in terms of the various conditional probabilities in the network.

The alarm and the failure of the alarm are d-separated from the blood sugar given the gauge.



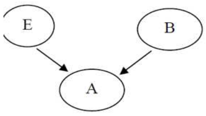

$$P(\text{BloodSugar} \mid \neg F_{BSG}, \text{BloodSugarGauge=high}) =$$

$$P(\text{BloodSugar}, \neg F_{BSG}, \text{BloodSugarGauge=high}) / P(\neg F_{BSG}, \text{BloodSugarGauge=high}) =$$

$$\alpha\ P(\text{BloodSugar}, \neg F_{BSG}, \text{BloodSugarGauge=high}) =$$

$$\alpha\ P(\text{BloodSugar})\, P(\neg F_{BSG}) P(\text{ BloodSugarGauge=high} \mid P(\text{BloodSugar}, \neg F_{BSG})$$

3. Assume a noisy Or gate model for p(A|E,B). Calculate the probability table by assuming p(¬A|E, ¬B) = 0.2 and p (¬A|¬E,B)=0.9.

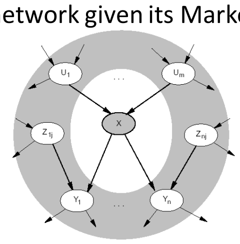



| E | B | A | ¬A |
|---|---|---|---|
| F | F | 0 | 1 |
| T | F | 0.8 | 0.2 |
| F | T | 0.1 | 0.9 |
| T | T | 0.82 | 0.2*0.9 |

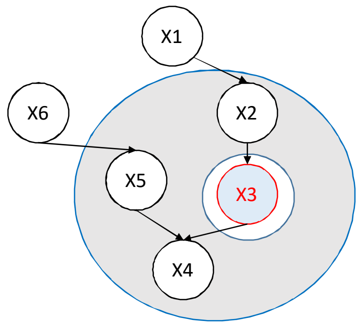


4. In the lecture, we defined the Markov blanket of a node:
   MB= parents + children + children's parents.
   Now, prove that a variable is independent of all other variables in the network given its Markov blanket.

   Find the simplest network to show how the general proof works.



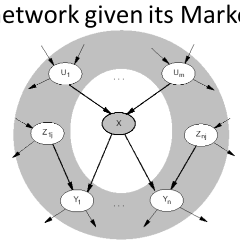
Simple example




$$P(x3|x1,x2,x4,x5,x6) = \frac{P(x1,x2,x3,x4,x5,x6)}{P(x1,x2,x4,x5,x6)}$$
$$= \frac{P(x1)*P(x2|x1)*P(x3|x2)*P(x4|x3,x5)*P(x5|x6)*P(x6)}{\sum_{x3} p(x1)*P(x2|x1)*P(x3|x2)*P(x4|x3,x5)*P(x5|x6)*P(x6)}$$
$$= \frac{P(x1)*P(x2|x1)*P(x3|x2)*P(x4|x3,x5)*P(x5|x6)*P(x6)}{P(x1)*P(x2|x1)*P(x5|x6)*P(x6)*\sum_{x3} P(x3|x2)*P(x4|x3,x5)}$$
$$= \frac{P(x3|x2)*P(x4|x3,x5)}{\sum_{x3} P(x3|x2)*P(x4|x3,x5)} = \alpha P(x3|x2)*P(x4|x3,x5)$$



4. In the lecture, we defined the Markov blanket of a node:
   MB= parents + children + children's parents.
   Now, prove that a variable is independent of all other variables in the network given its Markov blanket.



Suppose we prove it for a variable $x_i$ of n variables

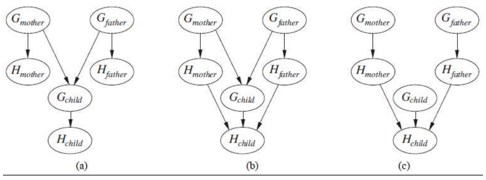
$$P(x_i|x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n) = \frac{P(x_1,\ldots,x_n)}{P(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n)}$$
$$= \frac{P(x_1,\ldots,x_n)}{\sum_{x_i} P(x_1,\ldots,x_n)}$$
$$= \frac{\prod_{j=1}^{n} P(x_j|parents(x_j))}{\sum_{x_i}\prod_{j=1}^{n} P(x_j|parents(x_j))}$$

Factor out all terms in the denominator that does not depend on $x_i$.
They also appear in the nominator, so delete them.

$$\frac{P(x_i|parents(x_i))\prod_{y_i \in children(x_i)} P(y_j|parents(y_j))}{\sum_{x_i} P(x_i|parents(x_i))\prod_{y_i \in children(x_i)} P(y_j|parents(y_j))}$$

$$= \alpha P(x_i|parents(x_i)) \prod_{y_i \in children(x_i)} P(y_j|parents(y_j))$$

5. Let $H_x$ be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values l or r, and perhaps actual handedness turns out to be the same as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability *m*, of a random mutation flipping the handedness.

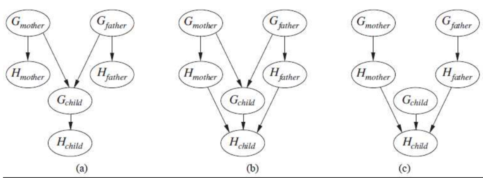



a. Which of the three networks in the following figure claim that
$\mathbf{P}(G_{father}, G_{mother}, G_{child}) = \mathbf{P}(G_{father})\mathbf{P}(G_{mother})\mathbf{P}(G_{child})$?

(c) matches the equation. The equation describes absolute independence of the three genes, which requires no links among them.



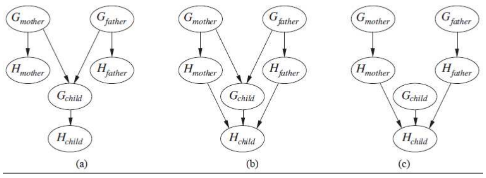
5. Let $H_x$ be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values l or r, and perhaps actual handedness turns out to be the same as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability *m*, of a random mutation flipping the handedness.




b. Which of the three networks make independence claims that are consistent with the hypothesis about the inheritance of handedness?

(a) and (b). The assertions are the absent links in c); the extra links in (b) may be unnecessary. (c) asserts independence of genes which contradicts the inheritance scenario.

5. Let $H_x$ be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values l or r, and perhaps actual handedness turns out to be the same as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability *m*, of a random mutation flipping the handedness.

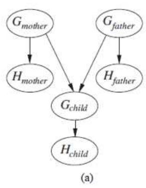



c. Which of the three networks is the best description of the hypothesis?

(a) is best



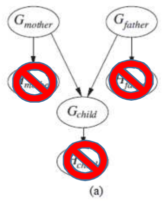
5. Let $H_x$ be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene $G_x$, also with values l or r, and perhaps actual handedness turns out to be the same as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability *m*, of a random mutation flipping the handedness.




d. Write down the CPT for the $G_{child}$ node in network (a).

| $G_{mother}$ | $G_{father}$ | $P(G_{child}=l\|...)$ | $P(G_{child}=r\|...)$ | |
|---|---|---|---|---|
| l | l | 1-m | m | 0.5*(1-m) + 0.5*(1-m)=1-m |
| l | r | 0.5 | 0.5 | 0.5*(1-m) + 0.5*(m)=0.5 |
| r | l | 0.5 | 0.5 | |
| r | r | m | 1-m | |

e. Suppose that $P(G_{father} = l) = P(G_{mother} = l) = \boldsymbol{q}$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of $\boldsymbol{m}$ and $\boldsymbol{q}$ only, by conditioning on its parent nodes.




| $G_{mother}$ | $G_{father}$ | $P(G_{child}=l\mid\ldots)$ | $P(G_{child}=r\mid\ldots)$ |
|---|---|---|---|
| l | l | 1-m | m |
| l | r | 0.5 | 0.5 |
| r | l | 0.5 | 0.5 |
| r | r | m | 1-m |

$$P(G_{child} = l) = \sum_{g_m, g_f} P(G_{child} = l \mid g_m, g_f) * P(g_m, g_f)$$

$$= \sum_{g_m, g_f} P(G_{child} = l \mid g_m, g_f) * P(g_m) * P(g_f)$$

$$= (1-m)q^2 + 0.5q(1-q) + 0.5(1-q)q + m(1-q)^2$$

$$\vdots$$

$$= q + m - 2mq$$



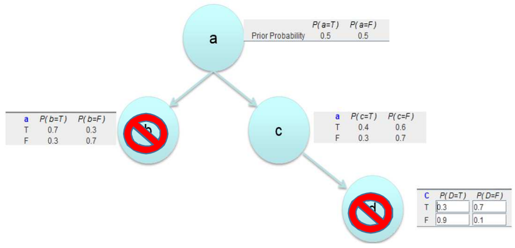
6. Name and explain criteria for simplifying BNs before evaluating a query.

d – separation → Bayes Ball

Irrelevant variables → Y is irrelevant unless Y ∈ Ancestors({Query variables} ∪ {Observations})

m – separation, uses the Markov Blanket property

7. Suppose you are given four Boolean random variables a, b, c and d and the following Bayesian Network




a. Write down an expression for computing the distribution of C, nothing given. Also, explain which variables can be eliminated.

$$P(C) = \alpha \sum_a P(a)P(C|a) \sum_b P(b|a) \sum_d P(d|C)$$

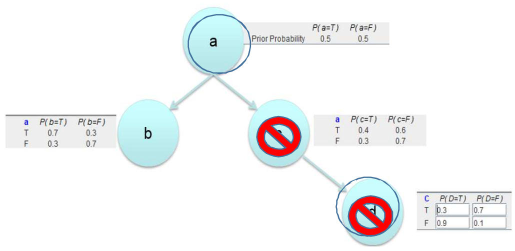
$$P(C) = \alpha \sum_a P(a)P(C|a) \quad = \alpha(0.5 < 0.4; 0.6 > +0.5 < 0.3; 0.7 >)$$

$$= \alpha(< 0.2; 0.3 > + < 0.15; 0.35 >)$$

$$= \alpha(< 0.35; 0.65 >) = < 0.35; 0.65 >$$

17

7. Suppose you are given four Boolean random variables a, b, c and d and the following Bayesian Network




b. Write down an expression for computing the probability distribution of b, given a=true and d=true. Also, explain which variables can be eliminated.

$$P(B) = < 0.7; 0.3 >$$

18

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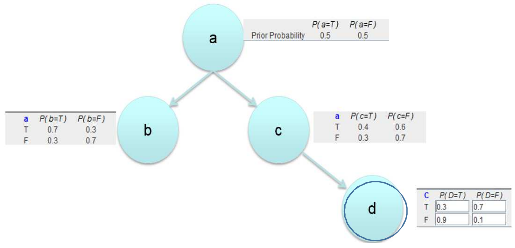


c. Write down an expression for computing the probability distribution of b given d=true. Also, explain which variables can be eliminated.

$$P(B) = \alpha \left( \sum_{a} P(a)P(B|a)P(c|a) \sum_{c} P\,(d|c) \right)$$



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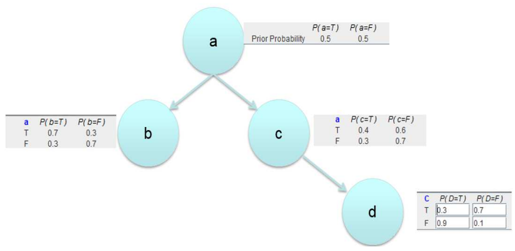


c. Write down an expression for computing the probability distribution of b given d=true. Also, explain which variables can be eliminated.

$$P(C) = \alpha \sum_{a} P(a)P(B|a) \sum_{c} P(c|a)P(d|c)$$