Intelligent Autonomous Agents and Cognitive Robotics
Exercise Sheet 3

1. Explain why it is a good heuristic to choose the variable that is most constrained but the value that is least constraining in a CSP search

2. Explain the main steps of the AC-3 algorithm. Check your approach with an example, e.g., arc consistency of the partial assignment {WA=green, V =red} for the map-colouring problem.

3. Would it be rational for an agent to hold the three beliefs $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \lor B) = 0.5$? If so, what range of probabilities would be rational for the agent to hold for $A \land B$? Make up a table and show how it supports your argument about rationality. Then draw another version of the table where $P(A \lor B) = 0.8$. Explain whether this is also consistent?

4. For each of the following statements, either proof it is true or give a counterexample.
   a. If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$

   b. If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$

   c. If $P(a|b) = P(a)$, then $P(a|b,c) = P(a|c)$

5. It is quite often useful to consider the effect of some specific propositions in the context of some general background evidence that remains fixed, rather than in the complete absence of information. The following questions ask you to prove more general versions of the product rule, with respect to some background evidence e. Prove the conditionalized version of the general product rule:
   $$P(A,B|E) = P(A|B,E)P(B|E) .$$

6. This exercise investigates the way in which conditional independence relationships affect the amount of information needed to for probabilistic calculations.
   - Suppose we wish to calculate $P(H| E_1, E_2)$ and we have no conditional independence information. Which of the following set of numbers are sufficient for the calculation?
     i. $P(E_1, E_2)$, $P(H)$, $P(E_1|H)$, $P(E_2|H)$
     ii. $P(E_1, E_2)$, $P(H)$, $P(E_1, E_2|H)$
     iii. $P(H)$, $P(E_1|H)$, $P(E_2|H)$
   - Suppose we know that $P(E_1| H, E_2) = P(E_1 | H)$ for all values of H, E_1, E_2. Now which of the three sets are sufficient?