Exercise 6

1. For the DBN of exercise 5 question 5 and for the evidence values

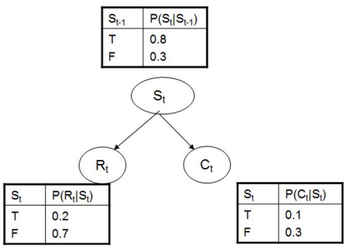
   e_1 = not red eyes, not sleeping in class

   e_2 = red eyes, not sleeping in class

   e_3 = red eyes, sleeping in class

   perform the following computation:

   a. Smoothing: P(*EnoughSleep*$_t$| $e_{1:3}$) for each of t=1,2,3
   b. Compare the filtered and smoothed probabilities for t=1 and t=2.



S_{t-1}	$P(S_t\|S_{t-1})$
T	0.8
F	0.3

S_t	$P(R_t\|S_t)$
T	0.2
F	0.7

S_t	$P(C_t\|S_t)$
T	0.1
F	0.3

The smoothed value can be computed by the forward and backward message

$$P(X_k|e_{1:t}) = \alpha f_k b_{k+1:t}$$

Last exercise we computed the forward message (state estimation) P(*EnoughSleep*$_t$ | $e_{1:t}$) for t = 1, 2, 3.

e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 = red eyes, sleeping in class

$$P(S_1|e_1) =< 0.864, 0.1357>$$
$$P(S_2|e_{1:2}) = < 0.510, 0.499 >$$
$$P(S_3|e_{1:3}) =< 0.1045, 0.8955 >$$

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a. First we compute the backward message

$$P(e_{k+1:t}|X_k) = \sum_{x_{k+1}} P(e_{k+1}|x_{k+1})P(e_{k+2}|x_{k+1})P(x_{k+1}|X_k)$$

S_{t-1}	$P(S_t\|S_{t-1})$
T	0.8
F	0.3

S_t	$P(R_t\|S_t)$
T	0.2
F	0.7

S_t	$P(C_t\|S_t)$
T	0.1
F	0.3

$$P(e_3|S_3) \;\; =< 0.2 * 0.1, 0.7 * 0.3 > \;\; =< 0.02, 0.21 >$$

$$P(e_3|S_2) = \sum_{s3} P(e_3|s_3)P(\quad|s_3)P(s_3|S_2) \;\; = [0.02 * 1 * \langle 0.8, 0.3\rangle, 0.21 * 1 * \langle 0.2, 0.7\rangle]$$

$$=< 0.02 * 0.8 + 0.21 * 0.2, 0.02 * 0.3 + 0.21 * 0.7 > = < 0.0588, 0.153 >$$

$$P(e_{2:3}|S_1) = \sum_{s2} P(e_2|s_2)P(e_3|s_2)P(s_2|S_1)$$

$$= [0.18 * 0.0588 * \langle 0.8, 0.3\rangle, 0.49 * 0.153 * \langle 0.2, 0.7\rangle]$$

$$= < 0.0233, 0.0556 >$$

e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 = red eyes, sleeping in class

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a. Now combine forward and backward messages

$$P(e_{k+1:t}|X_k) = \sum_{x_{k+1}} P(e_{k+1}|x_{k+1})P(e_{k+2}|x_{k+1})P(x_{k+1}|X_k)$$

S_{t-1}	$P(S_t\|S_{t-1})$
T	0.8
F	0.3

S_t	$P(R_t\|S_t)$
T	0.2
F	0.7

S_t	$P(C_t\|S_t)$
T	0.1
F	0.3

$P(e_3|S_2) = < 0.0588,0153 >$

$P(e_{2:3}|S_1) = < 0.0233,0.0556 >$

Smoothed estimate $P(X_k / e_{1:t}) = \alpha f_k b_{k+1:t}$

$P(S_1|e_{1:3}) = \alpha P(S_1|e_1)\, P(e_{2:3}|S_1)$

$= \alpha < 0.8643,0.1357 > < 0.0233,0.0556 >$

$= < 0.7277,0.273 >$

$P(S_2|e_{1:3}) = \alpha P(S_2|e_{1:2})\, P(e_3|S_2)$

$= \alpha < 0.5010,0.4990 > < 0.0588,0.153 >$

$= < 0.2757,0.7243 >$

$P(S_3|e_{1:3}) = < 0.1045, 0.8955 >$

e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 = red eyes, sleeping in class

Forward messages:

$P(S_1|e_1) =< 0.864,0.1357>$

$P(S_2|e_{1:2}) = < 0.510, 0.499 >$

$P(S_3|e_{1:3}) =< 0.1045,0.8955 >$

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b. Compare the filtered and smoothed probabilities for t=1 and t=2.

Filtering

$P(S_1|e_1) =< 0.864,0.1357>$

$P(S_2|e_{1:2}) = < 0.510, 0.499 >$

$P(S_3|e_{1:3}) =< 0.1045,0.8955 >$

e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 = red eyes, sleeping in class

Smoothed estimate

$P(S_1|e_{1:3}) = < 0.7277,0.273 >$

$P(S_2|e_{1:3}) = < 0.2757,0.7243 >$

$P(S_3|e_{1:3}) =< 0.1045,0.8955 >$

The smoothed analysis places the time the student started sleeping poorly one step earlier than the filtered analysis

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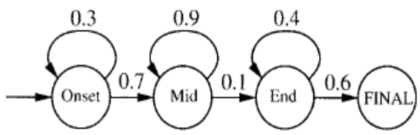
3. The following network represents the detection of the phone [m]. As you can see the states can produce different outputs with different probabilities. Calculate the most probable path for the following network and the output sequence using context information.
   (C1, C2, C3, C4, C4, C6, C7). Also give the total probability of the observation sequence



Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

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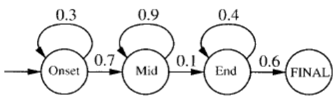
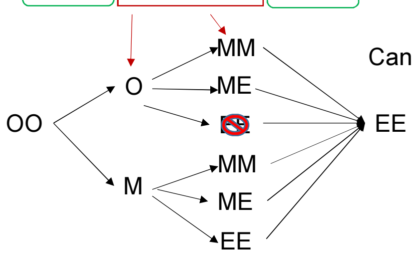


Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

The [C1,C2] must come from the **Onset**.
The [C6,C7] must come from the **End**.

(C1, C2, C3, C4, C4, C6, C7)



Can not go directly from **Onset** to **End**.

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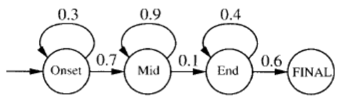


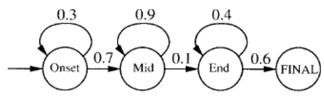
Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

These are all possible state sequences that can produce the observations.

O	O	O	M	M	E	E
O	O	O	M	E	E	E
O	O	M	M	M	E	E
O	O	M	M	E	E	E
O	O	M	E	E	E	E



Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

First question: What is the transition probability of the state sequence?

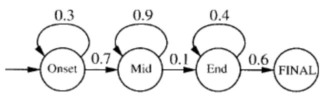
OOOMMEE (* .3 .3 .7 .9 .1 .4 .6

OOOMEEE (* .3 .3 .7 .1 .4 .4 .6

OOMMMEE (* .3 .7 .9 .9 .1 .4 .6

OOMMEEE (* .3 .7 .9 .1 .4 .4 .6

OOMEEEE (* .3 .7 .1 .4 .4 .4 .6



Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

Second question: What are the observation probabilities?

```
OOOMMEE (* .3 .3 .7 .9 .1 .4 .6 .5 .2 .3 .7 .7 .5 .4) = 4.0e-6
OOOMEEE (* .3 .3 .7 .1 .4 .4 .6 .5 .2 .3 .7 .1 .5 .4) = 2.5e-7
OOMMMEE (* .3 .7 .9 .9 .1 .4 .6 .5 .2 .2 .7 .7 .5 .4) = 8.0e-6
OOMMEEE (* .3 .7 .9 .1 .4 .4 .6 .5 .2 .2 .7 .1 .5 .4) = 5.1e-7
OOMEEEE (* .3 .7 .1 .4 .4 .4 .6 .5 .2 .2 .1 .1 .5 .4) = 3.2e-8
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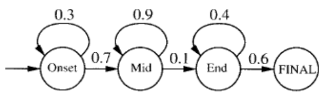

Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

What is the most likely sequence?

```
OOOMMEE (* .3 .3 .7 .9 .1 .4 .6 .5 .2 .3 .7 .7 .5 .4) = 4.0e-6
OOOMEEE (* .3 .3 .7 .1 .4 .4 .6 .5 .2 .3 .7 .1 .5 .4) = 2.5e-7
OOMMMEE (* .3 .7 .9 .9 .1 .4 .6 .5 .2 .2 .7 .7 .5 .4) = 8.0e-6
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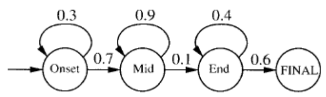

Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

What is the most likely sequence?

```
OOOMMEE (* .3 .3 .7 .9 .1 .4 .6 .5 .2 .3 .7 .7 .5 .4) = 4.0e-6
OOOMEEE (* .3 .3 .7 .1 .4 .4 .6 .5 .2 .3 .7 .1 .5 .4) = 2.5e-7
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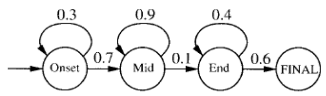

Output probabilities for the phone HMM:

Onset:	Mid:	End:
C_1: 0.5	C_3: 0.2	C_4: 0.1
C_2: 0.2	C_4: 0.7	C_6: 0.5
C_3: 0.3	C_5: 0.1	C_7: 0.4

(C1, C2, C3, C4, C4, C6, C7)

What is the total probability of the sequence?

```
OOOMMEE (* .3 .3 .7 .9 .1 .4 .6 .5 .2 .3 .7 .7 .5 .4) = 4.0e-6
OOOMEEE (* .3 .3 .7 .1 .4 .4 .6 .5 .2 .3 .7 .1 .5 .4) = 2.5e-7
OOMMMEE (* .3 .7 .9 .9 .1 .4 .6 .5 .2 .2 .7 .7 .5 .4) = 8.0e-6
OOMMEEE (* .3 .7 .9 .1 .4 .4 .6 .5 .2 .2 .7 .1 .5 .4) = 5.1e-7
OOMEEEE (* .3 .7 .1 .4 .4 .4 .6 .5 .2 .2 .1 .1 .5 .4) = 3.2e-8
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$\sum$ over all pathes $=$

$1.3\text{x}10^{-5}$

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