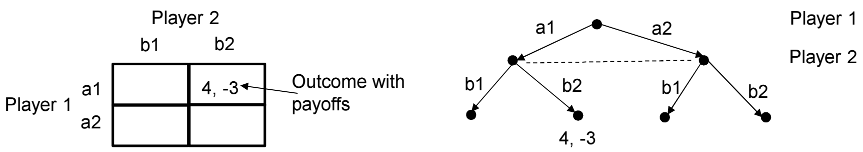
Exercise 10

1. Explain the difference between normal form and extensive form games.



In extensive form game representations one can see all the information as in normal form games. In addition an agent can also see which agent is on turn, what actions are available in each state, the information set if the history of actions is not known to the agent.

2. In Multi-Agent environments agents are trying to find a strategy that is optimal for them. This strategy might depend on the strategy decision of other agents. In this context explain the following concepts:
- Strategy
- Strategy Profile
- Dominant Strategy Equilibrium

A strategy is a contingency plan for all possible states of a game.

$s_i=(a_1,\dots,d_n)$

A strategy profile is the set of all strategies of all agents.

$s=(s_1,\dots,s_n)$

A strategy for a agent i is dominant, if it is a best response for all strategies of other agents.

$u_i(s_i^*,s_{-i})\geq u_i(s_i',s_{-i}) \quad \forall\, s_i' \quad \forall\, s_{-i}$

A dominant strategy equilibrium is a strategy profile where the strategy for each player is dominant (s_i^*).

$u_i(s_i^*,s_{-i})\geq u_i(s_i',s_{-i}) \quad \forall\, s_i' \quad \forall\, s_{-i} \quad \forall\, i$

3. Acme, a video game hardware manufacturer, has to decide whether its next game machine will use DVDs or CDs. Meanwhile, the video game software producer Best needs to decide whether to produce its next game on DVD or CD. The profits of both will be positive if they agree and negative if they disagree, as is shown in the following payoff matrix:

	Acme: dvd	Acme: cd
Best: dvd	A=9, B=9	A=-4, B=-1
Best: cd	A=-3, B=-1	A=5, B=5

Is there a dominant strategy? No dominant strategy equilibrium

Are there Nash equilibria? Two Nash equilibria

What is the Pareto-optimal solution? (dvd, dvd) they agree on this

What does happen if we change (dvd, dvd) to (A=5, B=5)?

Two pareto optimal solutions →
communicate (coordination game), define order of solutions before the game starts

4. Show that a dominant strategy equilibrium is a Nash equilibrium, but not vice versa.

Dominant strategy equilibrium: $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall\, s_i',\ \forall\, s_{-i},\ \forall\, i,$

Nash equilibrium: $u_i(s_i^*, s_{-i}) \geq u_i(s_i', s_{-i})\ \forall\, s_i', \qquad \forall\, i$

Dominant strategy equilibrium is a special case of Nash equilibrium.

5. The payoff matrix below, from Blinder (1983) by way of Bernstein (1996), shows a game between politicians and the Federal Reserve. Politicians can expand or contract fiscal policy, while the Fed can expand or contract monetary policy. And of course either side can choose to do nothing. Each side also has preferences for who should do what—neither side wants to look like the bad guys. The payoffs shown are simply the rank orderings; 9 for first choice through 1 for last choice. Find the Nash equilibrium of the game in pure strategies. Is this a Pareto optimal solution?

	Fed: contract	Fed: do nothing	Fed: expand
Pol: contract	~~F=7, P=1~~	~~F=9, P=4~~	~~F=6, P=6~~
Pol: do nothing	~~F=8, P=2~~	F=5, P=5	F=4, P=9
Pol: expand	F=3, P=3	F=2, P=7	F=1, P=8

Nash and Dominant strategy equilibrium

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Pol: expand	F=3, P=3	F=2, P=7	F=1, P=8

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Pol: do nothing	F=8, P=2	F=5, P=5	F=4, P=9
Pol: expand	F=3, P=3	F=2, P=7	F=1, P=8

NO

Every decision in this region is better.

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6. In the game three-finger Morra, two players, O (Odd) and E (Even), simultaneously display one, two or three fingers. Let the total numbers of fingers be **f**. If **f** is odd, O collects **f** dollars from E, and if **f** is even, E collects **f** dollars from O. Determine the best strategies for the players.

	O:1	O:2	O:3
E:1	2,-2	-3,3	4,-4
E:2	-3,3	4,-4	-5, 5
E:3	4,-4	-5,5	6,-6

6. Three finger Morra

	O:1	O:2	O:3
E:1	2,-2	-3,3	4,-4
E:2	-3,3	4,-4	-5,5
E:3	4,-4	-5,5	6,-6

What is the EU under the mixed strategy?

4-2r-7s = 4-1/2-7/2 = 0

O chooses for 1,2,3 the mixed strategy [r, s, (1-r-s)]

If E plays “1”, E’s expected utility is 2r-3s+4(1-r-s) = 4-2r-7s

If E plays “2”, E’s expected utility is -3r+4s-5(1-r-s) = -5+2r+9s

If E plays “3”, E’s expected utility is 4r-5s+6(1-r-s) = 6-2r-11s

Setting the first and third equation equal

4-2r-7s = 6-2r-11s => 0 = 2-4s =>s=1/2

Setting the second and third equation equal

-5+2r+9s = 6-2r-11s => 0 =11-4r-20s => 0 =11-4r-10 => 1-4r=0 => r=1/4

So O plays the mixed strategy [1/4, 1/2, 1/4]

6. Three finger Morra

	O:1	O:2	O:3
E:1	2,-2	-3,3	4,-4
E:2	-3,3	4,-4	-5,5
E:3	4,-4	-5,5	6,-6

What is the EU under the mixed strategy?

-4+2r+7s = -4+1/2+7/2 = 0

E chooses for 1,2,3 the mixed strategy [r, s, (1-r-s)]

If O plays "1", O's expected utility is -2r+3s-4(1-r-s) = -4+2r+7s

If O plays "2", O's expected utility is 3r-4s+5(1-r-s) = 5-2r-9s

If O plays "3", O's expected utility is -4r+5s-6(1-r-s) = -6+2r+11s

Setting the first and third equation equal

-4+2r+7s= -6+2r+11s => 0 = -2+4s =>s=1/2

Setting the second and third equation equal

5-2r-9s = -6+2r11s => 0 =-11+4r+20s => 0 =-11+4r+10 => -1+4r=0 => r=1/4

So E plays the mixed strategy [1/4, 1/2, 1/4]

Mixed strategy equilibrium is ([1/4, 1/2, 1/4], ([1/4, 1/2, 1/4]) 11

What about two finger Morra and EU?

	O:1	O:2
E:1	2,-2	-3,3
E:2	-3,3	4,-4

O chooses [p, (1-p)]

2p-3(1-p) = -3p+4(1-p)

5p-3 = 4-7p → p = 7/12 4-7p=4-7/2=1/2

E chooses [q, (1-q)]

-2q+3(1-q)= 3q-4(1-q)

-5q+3 = 7q - 4 → q = 7/12 7q – 4 = 7/2-4= -1/2

The mixed Nash Equilibrium is <[7/12,5/12], <[7/12,5/12],>

So, it is better to be the Odd player.

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7. There are three candidates or choices A, B and C. The voters have to define a preference list. The results are given below:

Rang	u	v	w	x	y	z
1	A	A	B	B	C	C
2	B	C	A	C	A	B
3	C	B	C	A	B	A

u persons like A more then B and B more then C, v persons have the preference list ACB, w persons have the preference list BAC, etc.
Give general conditions on the numbers for a win of A.
Use the Condorcet Criteria.

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7. Use the Condorcet Criteria.

Rang	u	v	w	x	y	z
1	A	A	B	B	C	C
2	B	C	A	C	A	B
3	C	B	C	A	B	A

A wins if
A>B u+v+y > w+x+z
A>C u+v+w > x+y+z

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7. Use the Condorcet Criteria.

Rang	u			u	u	
1	A			B	C	
2	B			C	A	
3	C			A	B	

Discuss the case: $u = x = y$ and $v = w = z = 0$.

A wins if

A>B $u+v+y > w+x+z$

A>C $u+v+w > x+y+z$

$2u > u \rightarrow A>B$

$u < 2u \rightarrow C>A$

Have to compare B and C $2u>u \rightarrow B>C$

The values $u = x = y$ und $v = w = z = 0 \rightarrow$ NO Condorcet winner

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