Assignment 10

Discussed during the tutorial on January 19th, 2023

10.1 (10 points) Consider the IP

$$\max x_1 + 10x_2 \text{ subject to } \begin{pmatrix} 1 & -20 \\ 1 & 20 \end{pmatrix} x \leq \begin{pmatrix} 0 \\ 20 \end{pmatrix}, x \geq 0, x \in \mathbb{Z} \ .$$

(a) What is the minimum Euclidean distance, in $\mathbb{R}^2$, between the optimal solution of the IP and of its LP relaxation? Explain.

(b) Given an example of an IP for which the minimum Euclidean distance between its optimal solution and that of its LP relaxation is at least 100.

(c) Give an example of an IP which has no feasible integer solutions, but its LP relaxation has a feasible set in $\mathbb{R}^2$ of area at least 100.

10.2 (10 points) Consider the IP problem

$$\max\{x_1 + 6x_2 \mid x_1 + 9x_2 \leq 10, 2x_1 \leq 19, x \geq 0, x \in \mathbb{Z}\} \ .$$

(a) Graph the feasible region of the LP relaxation of the IP; indicate the feasible solutions of the IP on the graph. Then find the optimal solutions of the LP relaxation and the IP.

(b) Focus on the optimal solution of the LP relaxation. What are the closest (in the sense of Euclidean distance) feasible solutions of the IP? Are any of these closest feasible IP solutions optimal for the IP? Now replace the objective function with $\max x_1 + 8.5x_2$. Answer the same questions again. What related conclusions can you draw from these examples?

(c) Create a family of IP problems with two non-negative integer variables parameterized by an integer $k \geq 10$ (in the above example $k = 10$) generalizing the above example and its highlighted features. Then answer parts (a) and (b) for all values of k.

10.3 (10 points) Suppose that we solve an LP relaxation of a pure IP and that for the optimal basis $B = \{1, 2, 3\}$ of (P), we have the following canonical form:

$$\begin{aligned} \max \quad & \left(0, 0, 0, \frac{17}{12}, \frac{1}{12}, \frac{5}{12}\right) x + 140 \\ \text{subject to} \quad & \begin{pmatrix} 1 & 0 & 0 & \frac{111}{60} & -\frac{13}{12} & -\frac{1}{60} \\ 0 & 1 & 0 & \frac{1}{10} & \frac{1}{2} & -\frac{1}{10} \\ 0 & 0 & 1 & 2 & \frac{11}{2} & -\frac{1}{12} \end{pmatrix} x = \begin{pmatrix} \frac{9}{2} \\ 1 \\ \frac{11}{10} \end{pmatrix} \\ & x \geq 0 \ . \end{aligned}$$

Indicate which constraints lead to cutting planes and for each such constraint generate the corresponding cutting plane.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on 17th of January.