Assignment 3

Discussed during the tutorial on Thursday, November 10th, 2022

3.1 (10 points) You are about to trek across the desert with a vehicle having 3.6 cubic metres of cargo space for goods. There are various types of items available for putting in this space, each with a different volume and a different net value for your trip, shown as below:

Item type i	1	2	3	4	5	6	7
Volume v_i in m^3	0.55	0.60	0.70	0.75	0.85	0.90	0.95
Net value n_i in €	250	300	500	700	750	900	950

(a) You need to decide which items to take, not exceeding the volume constraint. You can take at most one of any item type. No item can be split into fractions. The total net value must be maximized. Formulate this problem as an LP or an IP. You may use the notation v_i for volume and n_i for net value of item i.

(b) Your two friends have decided to come as well, each with an identical vehicle as yours. In total, there are exactly two items of each type available. The question is whether you can fit all 14 items in the vehicles without exceeding the volume constraints. No cutting of items into pieces is permitted. Each item taken must be placed entirely into one of the vehicles. Formulate an LP or IP that has a feasible solution if and only if the items can be packed as desired. Note that the net value is ignored for this part of the question.

3.2 (10 points) Consider an LP with variables x_1, x_2, x_3, x_4. Suppose that the LP includes the constraints that $x_1, x_2, x_3, x_4 \geq 0$.

(a) Consider the constraint $x_4 \geq |x_3 - 2x_1|$. Suppose that we want to add to the LP the condition that this constraint is satisfied. Show how to satisfy this requirement so that the resulting formulation is an LP.
Hint: rewrite the constraint as a pair of linear inequalities.

(b) Consider the following inequalities:

$$6x_1 + 2x_2 + 3x_3 + 3x_4 \geq 3,$$
$$2x_1 + 4x_2 + 2x_3 + 7x_4 \geq 9 \ .$$

Suppose that we want to add to an IP the condition that at least one of these two constraints is satisfied. Show how to satisfy this requirement so that the resulting formulation is an IP.
Hint: add a binary variable indicating which of the two constraints must be satisfied.

(c) Suppose that for $i = 1, \ldots, k$ we have a non-negative vector a^i with four entries and a number β_i (both a^i and β_i are constants). Let r be any number between 1 and k. Consider the following set of inequalities:

$$(a^i)^T x \geq \beta_i, \qquad i = 1, \ldots, k \ . \tag{1}$$

We want to add to an IP the condition that at least r of the constraints are satisfied. Show how to satisfy this requirement so that the resulting formulation is an IP.
Hint: add a binary variable for each constraint in (1).

(d) Consider the following set of values, $S = \{3, 9, 17, 19, 36, 67, 1893\}$. Suppose that we want to add to an IP the condition that the variable x takes only one of the values in S. Show how to satisfy this requirement so that the resulting formulation is an IP.
Hint: add a binary variable for each number in the set S.

3.3 (10 points) The company HHTech won a bid by the Free and Hanseatic City of Hamburg to meet the yearly demands $d_1, \dots, d_n$ in the city areas $j = 1, \dots, n$. Now the company has to decide where to build its factories and how much of each factory's output will be shipped to each of the n areas.

There are m potential locations for building the factories. If the company decides to build at location i, for $i \in \{1, \dots, m\}$, then the fixed cost of building the factory (yearly amortized version) is f_i and the yearly capacity of the factory will be s_i. The cost of transporting one unit of the product from location i to area j is given as c_{ij}.

Construct an IP whose solution indicates where to build the factories, how many units of product to ship from each factory to each demand area so that the demand is met and the total yearly cost of the company is minimized.

3.4 (10 points) A 9×9 matrix A is partitioned into 3×3 submatrices $A_1, \dots, A_9$ of consecutive elements. Certain entries of A contain numbers from the set $\{1, \dots, 9\}$. A solution to the Sudoku game is an assignment of integers from 1 to 9 to each (unassigned) entry of the matrix such that

- each row of A,
- each column of A,
- each submatrix $A_1, \dots, A_9$

contains every number from $\{1, \dots, 9\}$ exactly once. Formulate this problem of checking whether there is a solution to the Sudoku game as an integer feasibility problem, i.e., as an IP without objective function.
Hint: define a binary variable x_{ijk} that takes value 1 when entry (i, j) is assigned value k.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until the 8am on Tuesday, 8th of November.