Assignment 11

Discussed during the tutorial on January 26th

10.1 (10 points) Consider the following NLP:

$$\min\ x_3 \text{ s.t. } x_1 + x_2 \leq 0, x_1^2 - 4 \leq 0, x_1^2 - 2x_1 + x_2^2 - x_3 + 1 \leq 0\ .$$

Let $\bar{x} = (1/2, -1/2, 1/2)^T$. Write down the optimality conditions for $\bar{x}$ for this NLP as described in the Karush-Kuhn-Tucker theorem. Using these conditions and the theorem, prove that $\bar{x}$ is optimal.

10.2 (10 points) Let $u, w \in \mathbb{R}^n$ be given such that u_j and w_j are positive for each j. Consider the following NLP:

$$\min\ -\sum_{j=1}^{n} w_j \ln(x_j) \text{ s.t. } u^T x = n, -x \leq 0\ .$$

(a) Prove that this NLP is convex.

(b) Using the Karush-Kuhn-Tucker theorem (on possibly a slight modification of the NLP), find an optimal solution in terms of u and w.

(c) Prove that the solution you found is the unique optimal solution.

10.3 (10 points) Consider the following NLP:

$$\min\ -7x_1 - 5x_2 \text{ s.t. } 2x_1^2 + x_2^2 + x_1 x_2 - 4 \leq 0, x_1^2 + x_2^2 - 2 \leq 0, -x_1 + 1/2 \leq 0\ .$$

Let $\bar{x} = (1, 1)^T$. Write down the optimality conditions for $\bar{x}$ for this NLP described in the Karush-Kuhn-Tucker theorem. Using these conditions and the theorem, prove that $\bar{x}$ is optimal. Note, you may use the fact that the functions defining the objective function and the constraints are convex and differentiable without proving it.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on 24th of January.