Assignment 9

Discussed during the tutorial on December 22nd, 2022

9.1 Let (P) denote the following linear programming problem

$$\begin{array}{ll} \text{minimize} & 4x_2 + 2x_3 \\ \text{subject to} & x_1 + x_2 + 3x_3 \leq 1 \\ & x_1 - 2x_2 + x_3 \geq 1 \\ & x_1 + 3x_2 - 6x_3 = 0 \\ & x_1, x_3 \geq 0 \\ & x_2 \text{ free} \end{array}$$

Use complementary slackness conditions to determine if $x = (3/5, -1/5, 0)$ is an optimal solution to (P).

9.2 Let (P) denote the following linear programming problem

$$\begin{array}{ll} \text{minimize} & x_1 + 2x_2 - 3x_3 \\ \text{subject to} & x_1 + 2x_2 + 2x_3 = 2 \\ & -x_1 + x_2 + x_3 = 1 \\ & -x_1 + x_2 - x_3 \geq 0 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Use complementary slackness conditions to determine if $x = (0, 1, 0)$ is an optimal solution to (P).

9.3 Prove the following theorem. Use duality for one direction

Theorem 1 (Farkas' Lemma) *Let $\boldsymbol{A}$ be a matrix of dimensions $m \times n$ and let $\boldsymbol{b}$ be a vector in $\mathbb{R}^n$. Then, exactly one of the following two alternatives holds:*

(a) There exists some $\boldsymbol{x} \geq 0$ such that $\boldsymbol{Ax} = \boldsymbol{b}$.

(b) There exists some vector $\boldsymbol{p}$ such that $\boldsymbol{p}'\boldsymbol{A} \geq \boldsymbol{0}$ and $\boldsymbol{p}'\boldsymbol{b} < 0$.

9.4 Prove the following theorem.

Theorem 2 *Suppose that the system of linear inequalities $\boldsymbol{Ax} \leq \boldsymbol{b}$ has at least one solution, and let d be some scalar. Then, the following are equivalent:*

(a) Every feasible solution to the system $\boldsymbol{Ax} \leq \boldsymbol{b}$ satisfies $\boldsymbol{c}'\boldsymbol{x} \leq d$.

(b) There exists some $\boldsymbol{p} \geq \boldsymbol{0}$ such that $\boldsymbol{p}'\boldsymbol{A} = \boldsymbol{c}'$ and $\boldsymbol{p}'\boldsymbol{b} \leq d$.

⋆⋆⋆ We wish you and your families happy holidays, and a peaceful end of the year. ⋆⋆⋆

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on December, 20th.