Linear and Nonlinear Optimization
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Assignment 2

Discussed during the tutorial on Thursday, November 3rd, 2022

2.1 (10 points) You wish to build a house and you have divided the process into a number of tasks, namely:

    B. excavation and building the foundation,

    F. raising the wooden frame,

    E. electrical wiring,

    P. indoor plumbing,

    D. dry walls and flooring,

    L. landscaping.

You estimate the following duration for each of the tasks (in weeks):

Task	B	F	E	P	D	L
Duration	3	2	3	4	1	2

Some of the tasks can only be started when some other tasks are completed. For instance, you can only build the frame once the foundation has been completed, i.e., task F can start only after task B is completed. All the precedence constraints are summarized as follows:

- F can start only after B is completed,
- L can start only after B is completed,
- E can start only after F is completed,
- P can start only after F is completed,
- D can start only after E is completed,
- D can start only after P is completed.

The goal is to schedule the starting time of each task such that the entire project is completed as soon as possible.

Formulate this problem as a linear program (LP). Explain your formulation. Note, there is no limit on the number of tasks that can be done in parallel.

Hint: introduce variables to indicate the times that the tasks start.

2.2 (10 points) The director of the HHTech startup needs to decide what salaries to offer its employees for the coming year. In order to keep the employees satisfied, she needs to obey the following constraints:

- Tom wants at least €20,000 or he will quit;
- Peter, Nina and Samir each want to be paid at least €5,000 more than Tom;
- Gary wants his salary to be at least as high as the combined salary of Tom and Peter;
- Linda wants her salary to be €200 more than Gary's salary;

- the combined salary of Nina and Samir should be at least twice the combined salary of Tom and Peter;
- Bob's salary is at least as high as that of Peter and at least as high as that of Samir;
- the combined salary of Bob and Peter should be at least €60,000;
- Linda should not make more money than the combined salary of Bob and Tom.

(a) Write an LP that will determine the salaries for the employees of HHTech that satisfy each of these constraints while minimizing the total salary expenses.

(b) Write an LP that will determine salaries for the employees of HHTech that satisfy each of these constraints while minimizing the salary of the highest paid employee.

(c) What is the relation between the solutions for (a) and (b)?

2.3 (10 points) Consider the following setup: we have factories 1 through m and stores 1 through n. Each factory i produces u_i units of a commodity and each store j requires ℓ_j units of that commodity. Note, each factory produces the same commodity, and each store requires the same commodity. The goal is to transfer the commodity from the factory to the stores. All the commodities going from the factories to the stores are first sent to one of two central storage hubs, A and B. The cost of transporting one unit of commodity from factory i to hub A (hub B) is given by a_i (resp. b_i). The cost of transporting one unit of commodity from hub A (hub B) to store j is given by a'_j (resp. b'_j). Here, the quantities $u_i, \ell_j, a_i, b_i, a'_j, b'_j$ are constants for all i and j. The problem is to decide how much to send from each factory to each hub and how much to send from each hub to each store so that each store receives the amount of commodity it requires, no factory sends out more commodity it produces, and such that the total transportation cost is minimized. Formulate this problem as an LP; you may assume that the number of units of commodity sent can be fractional.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on Tuesday, November 1st, 2022.