Exercise 1

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1 Exercise 1.1

$$\begin{array}{lrl} \text{minimize} & 0.14 * ca + 0.12 * p + 0.2 * br + 0.75 * ch + 0.15 * bu & \\ \text{subject to} & 23 * ca + 171 * p + 65 * br + 112 * ch + 188 * bu & \geq 2000 \\ & 0.1 * ca + 0.2 * p + 0 * br + 9.3 * ch + 16 * bu & \geq 50 \\ & 0.6 * ca + 3.7 * p + 2.2 * br + 7 * ch + 7.7 * bu & \geq 100 \\ & 6 * ca + 30 * p + 30 * br + 0 * ch + 2 * bu & \geq 250 \\ & ca, p, br, ch, bu & \geq 0 \end{array}$$

The lowest price is 2.317754919499106, provided by a combination of 7.714669051878356 baked potatoes and 9.279964221824688 servings of peanut butter.

2 Exercise 1.2

2.1 a)

Proof by contradiction: Because of $|\sum a_{ij} * x_j - bi| \leq y_i$ is the upper bound the x vector has to fulfill. $\sum y_i$ is minimal, if $A * x_1$ is close to b, if there were a different vector x_2, which would result in a smaller deviation to b the bound y_2 could also be smaller.

2.2 b)

The problem is the absolute value, instead it can be represented with 2 constraints.

$$
\begin{array}{lrl}
\text{minimize} & y_1 + y_2 + y_3 + y_4 & \\
\text{subject to} & 2*(x_1-100) - 1*(x_2-100) + 1 & \le y_1 \\
& -2*(x_1-100) + 1*(x_2-100) - 1 & \le y_1 \\
& 1*(x_1-100) + 1*(x_2-100) - 1 & \le y_2 \\
& -1*(x_1-100) - 1*(x_2-100) + 1 & \le y_2 \\
& 1*(x_1-100) + 3*(x_2-100) - 4 & \le y_3 \\
& -1*(x_1-100) - 3*(x_2-100) + 4 & \le y_3 \\
& -2*(x_1-100) + 4*(x_2-100) - 3 & \le y_4 \\
& 2*(x_1-100) - 4*(x_2-100) + 3 & \le y_4 \\
& x, y & \ge 0
\end{array}
$$

x = (100.16666666666663, 100.83333333333331)
therefore the solution is x $\approx (0.17, 0.83)$

2.3 c)

The new deviation function only restricts the largest deviation value, not the total.

$$
\begin{array}{lrl}
\text{minimize} & y & \\
\text{subject to} & -1-2*(x_1-100) + 1*(x_2-100) & \le y \\
& +1+2*(x_1-100) - 1*(x_2-100) & \le y \\
& +1-1*(x_1-100) - 1*(x_2-100) & \le y \\
& -1+1*(x_1-100) + 1*(x_2-100) & \le y \\
& +4-1*(x_1-100) - 3*(x_2-100) & \le y \\
& -4+1*(x_1-100) + 3*(x_2-100) & \le y \\
& +3+2*(x_1-100) - 4*(x_2-100) & \le y \\
& -3-2*(x_1-100) + 4*(x_2-100) & \le y \\
& x, y & \ge 0
\end{array}
$$

The result is x = (100.30434782608697, 101.04347826086956).
The solution is x $\approx (0.3, 1.0)$