Assignment 6

Discussed during the tutorial on Thursday, December 1st, 2022

6.1 (10 points) Consider the system $Ax = b$ where the rows of A are linearly independent. Let $\bar{x}$ be a solution to $Ax = b$. Let J be the set of column indices j of A for which $\bar{x}_j \neq 0$.

(a) Show that if $\bar{x}$ is a basic solution, then the columns of A_J are linearly independent.

(b) Show that if the columns of A_J are linearly independent, then $\bar{x}$ is a basic solution for some basis $B \supseteq J$.

Note that (a) and (b) give you a way of checking whether $\bar{x}$ is a basic solution, namely you simply need to verify whether the columns of A_J are linearly independent.

(c) Consider the system of equations

$$\begin{pmatrix} 1 & 1 & 0 & 2 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 & 0 & -2 & 1 \\ 1 & 2 & 1 & 5 & 4 & 3 & 3 \end{pmatrix} x = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$$

and the following vectors:

(i) $(1, 1, 0, 0, 0, 0, 0)^T$
(ii) $(2, -1, 2, 0, 1, 0, 0)^T$
(iii) $(1, 0, 1, 0, 1, 0, 0)^T$
(iv) $(0, 0, 1, 1, 0, 0, 0)^T$
(v) $(0, 1/2, 0, 0, 1/2, 0, 1)^T$

For each vector in (i)-(v), indicate if it is a basic solution or not. Please justify your answers.

(d) Which of the vectors in (i)-(v) are basic feasible solutions?

6.2 (10 points) Consider the LP $\max\{c^T x \mid Ax = b, x \geq 0\}$ where

$$A = \begin{pmatrix} 1 & 2 & -2 & 0 \\ 0 & 1 & 3 & 1 \end{pmatrix}, b = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, c = \begin{pmatrix} 0 \\ 3 \\ 1 \\ 0 \end{pmatrix}.$$

(a) Beginning with the basis $B = \{1, 4\}$, solve the problem. At each step, choose the entering variable and leaving variable by Bland's rule.

(b) Give a certificate of optimality or unboundedness for the problem, and verify it.

6.3 (10 points) The princess' wedding ring can be made from four types of gold 1,2,3,4 with the following amounts of milligrams of impurity per gram:

Type	1	2	3	4
mg of lead	1	2	2	1
mg of cobalt	0	1	1	2
value	1	2	3	2

Set up an LP which will determine the most valuable ring that can be made containing at most 6 mg of lead and at most 10 mg of cobalt. Put the LP into SEF and then solve it.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on Tuesday, November 29th.