Assignment 1

Discussed during the tutorial on Thursday, October 27th, 2022

1.1 Consider the following table indicating the nutritional value of different food types:

Food	Price (€) per serving	Calories per serving	Fat (g) per serving	Protein (g) per serving	Carbohydrate (g) per serving
Raw carrots	0.14	23	0.1	0.6	6
Baked potatoes	0.12	171	0.2	3.7	30
Wheat bread	0.2	65	0	2.2	13
Cheddar cheese	0.75	112	9.3	7	0
Peanut butter	0.15	188	16	7.7	2

You need to decide how many servings of each food to buy each day so that you minimize the total cost of buying your food while satisfying the following daily nutritional requirements:

- calories must be at least 2000,
- fat must be at least 50g,
- protein must be at least 100g,
- carbohydrates must be at least 250g.

Write an LP that will let you decide how many (fractional) servings of each of the aforementioned foods are needed to meet all the nutritional requirement, while minimizing the total cost of the food.

1.2 You are given a matrix $A \in \mathbb{R}^{m\times n}$ and a vector $b \in \mathbb{R}^m$ for which the system $Ax = b$ has no solution; for example:

$$\begin{aligned} 2x_1 - x_2 &= -1, \\ x_1 + x_2 &= 1, \\ x_1 + 3x_2 &= 4, \\ -2x_1 + 4x_2 &= 3 \ . \end{aligned}$$

We are interested in finding a vector $x \in \mathbb{R}^n$ that "comes close" to solving the system. Namely, we want to find an $x \in \mathbb{R}^n$ whose *deviation* is minimum, where the deviation of x is $|\sum_{i=1}^n |b_i - \sum_{j=1}^n a_{ij}x_j|$. For the example system above, the vector $x = (1,1)^T$ has deviation $2+1+0+1 = 4$.

(a) Show that the solution to the optimization problem

$$\min \sum_{i=1}^m y_i \text{ subject to } |\sum_{j=1}^n a_{ij}x_j - b_i| \leq y_i, \qquad i = 1,\ldots,m,$$

will give a vector x of minimum deviation.

(b) The problem of part (a) is not an LP. (Why?) Show that it can be formulated as an LP.

(c) Suppose that we had instead defined the deviation of x as $\max_{i=1,\ldots,m} |b_i - \sum_{j=1}^{n} a_{ij} x_j|$. (According to this definition, in the example above $x = (1,1)^T$ would have deviation $\max(2,1,0,1) = 2$.) With this new definition, write the problem of finding a vector of minimum deviation as an optimization problem, and show that this problem can also be formulated as an LP.

Upload your solutions as a `.pdf`-file to the course page on the TUHH e-learning portal until 8am on Tuesday, October 25th, 2022.