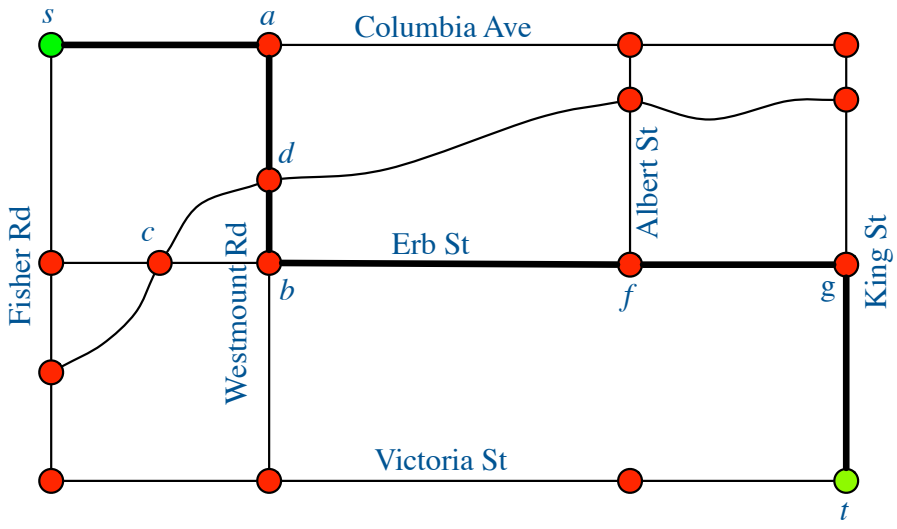
# Module 1: Formulations (Optimization on Graphs)

# Shortest Paths




- Familiar problem: Starting at location $s$, we wish to travel to $t$. What is the best (i.e., shortest) route?

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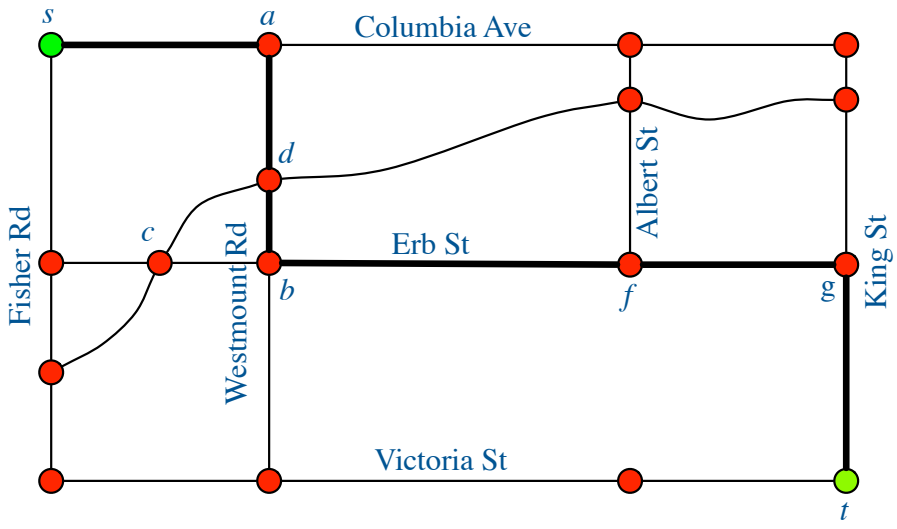


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- In the figure above, such a route is indicated in bold.

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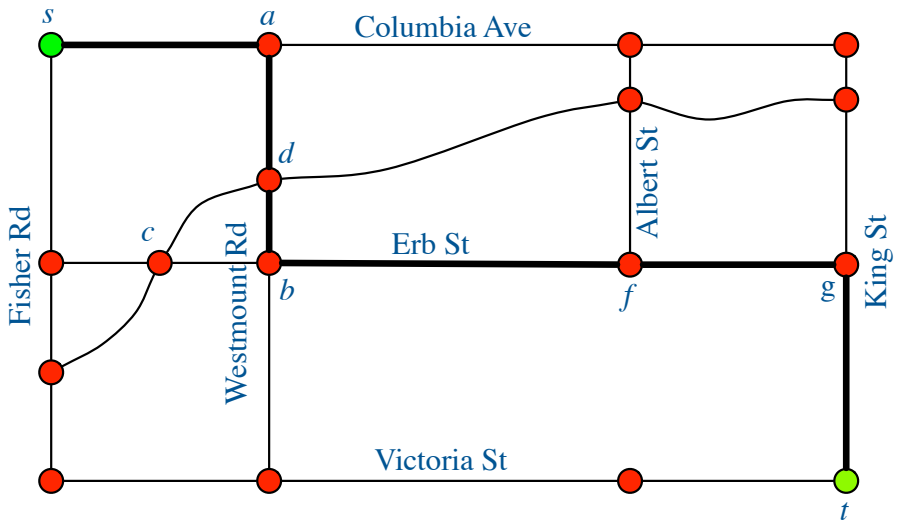


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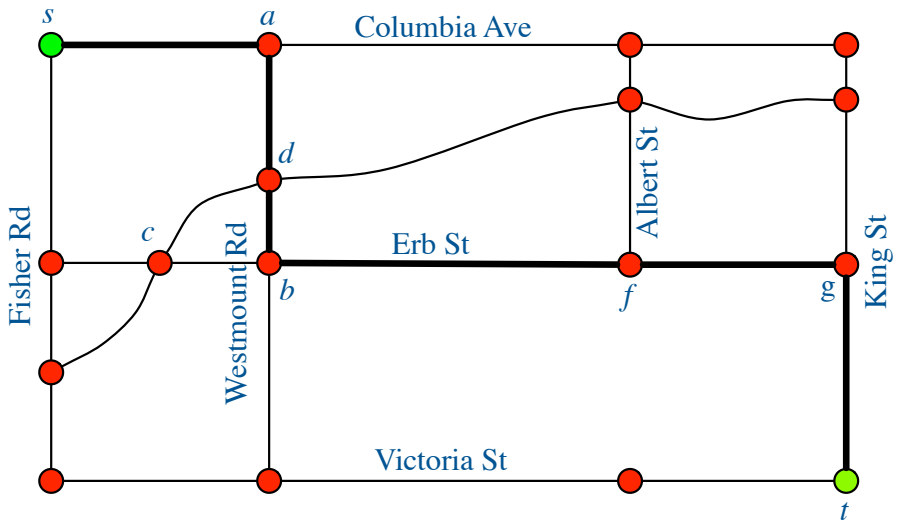


- Goal: Write the problem of finding the shortest route between $s$ and $t$ as an integer program!
  ... How?

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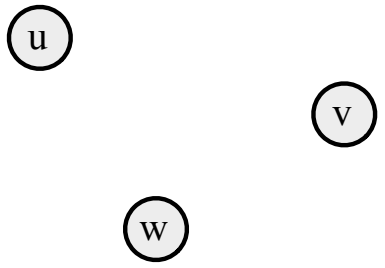

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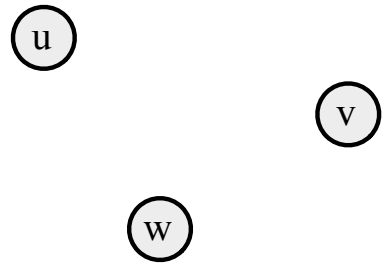

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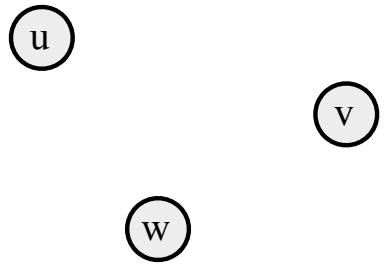

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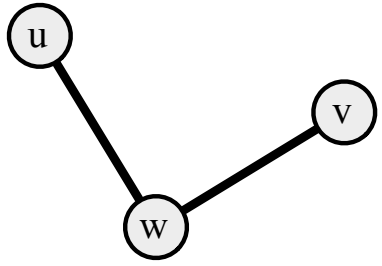

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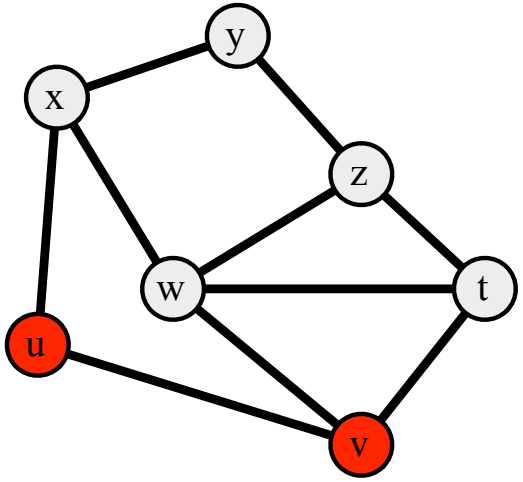

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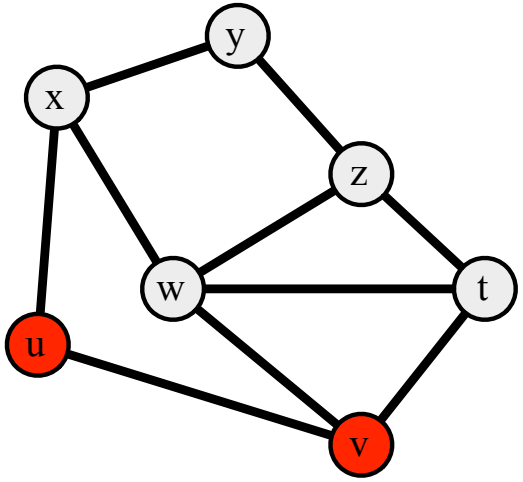

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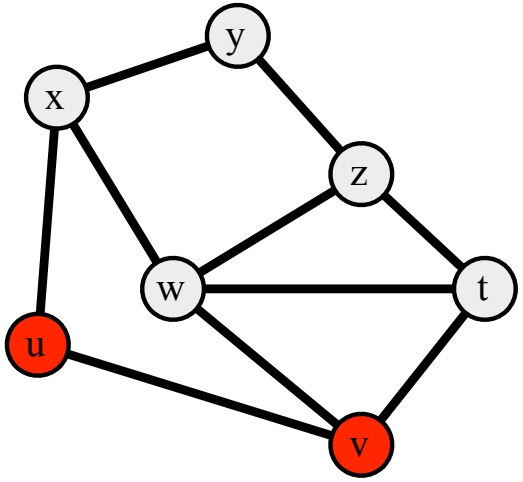

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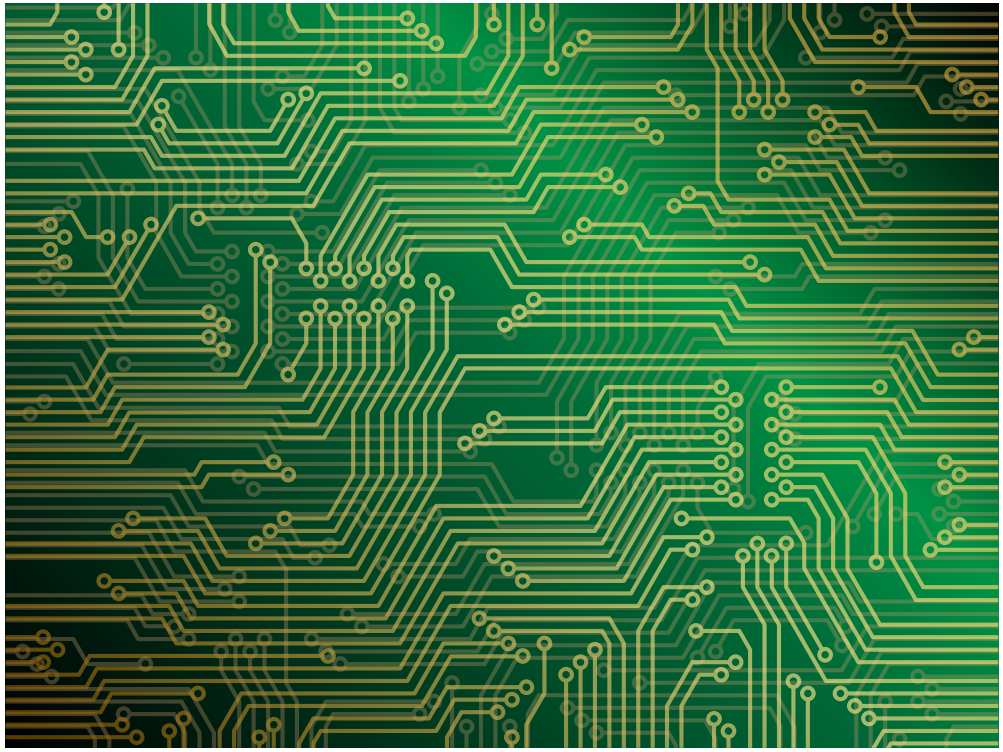

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VLADGRIN/iStock/Thinkstock

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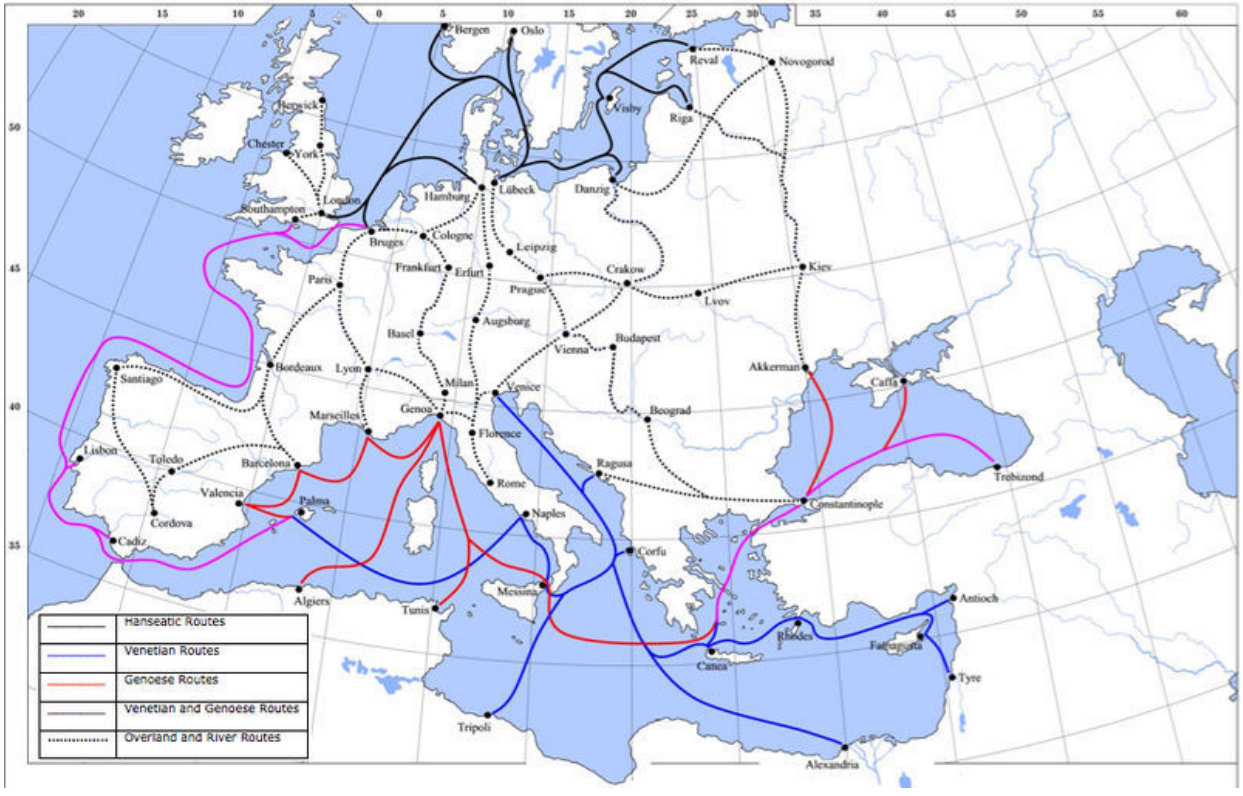


Lampman, 2008 [Online Image]. Late Medieval Trade Routes. Wikimedia Commons. http://commons.wikimedia.org/wiki/File:Late_Medieval_Trade_Routes.jpg

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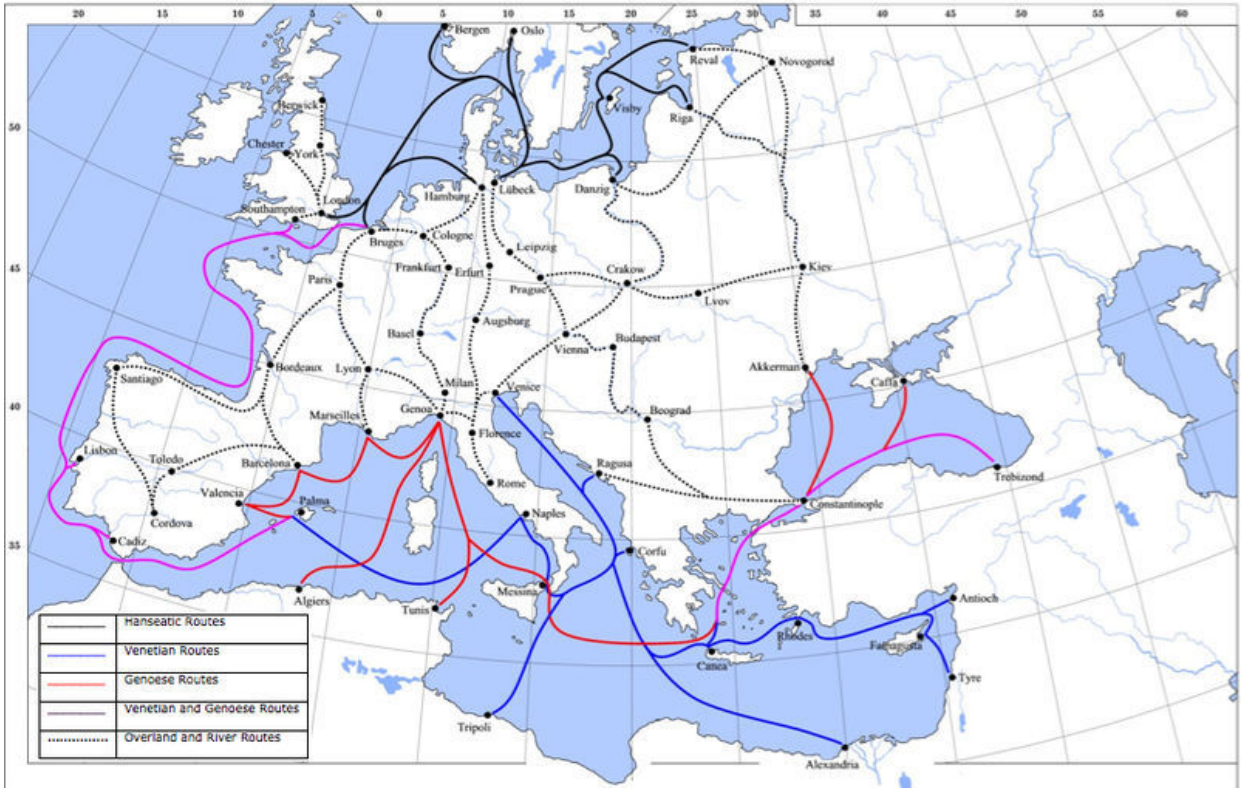
- Social networks
- Trade networks
- .... and many more!

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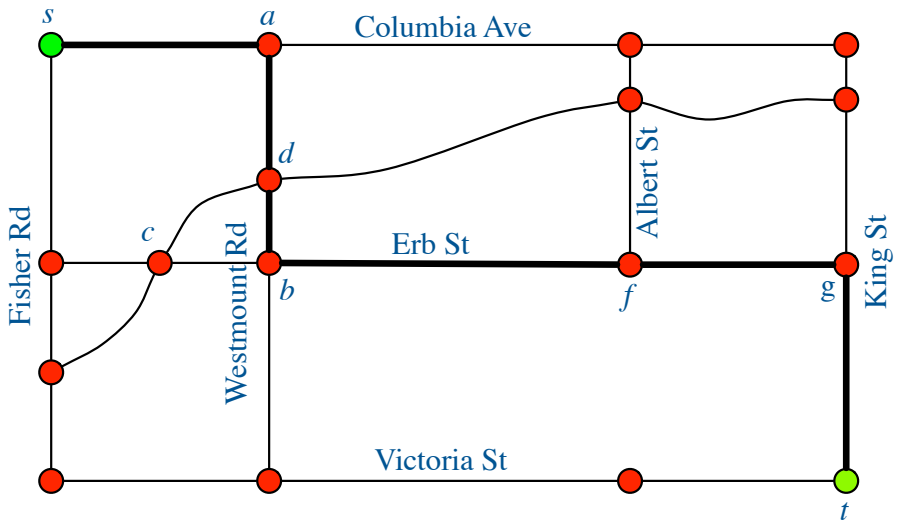
# The Map of Water Town

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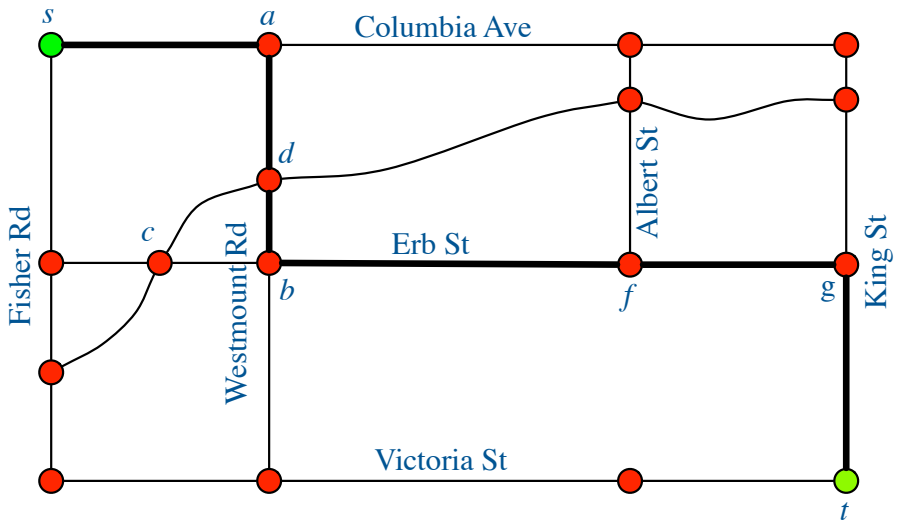


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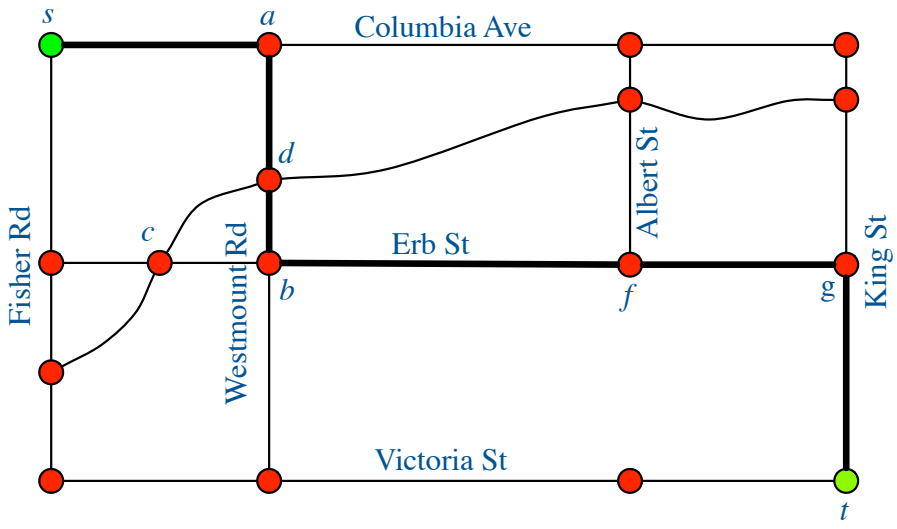


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- Edges: Road segments connecting adjacent intersections

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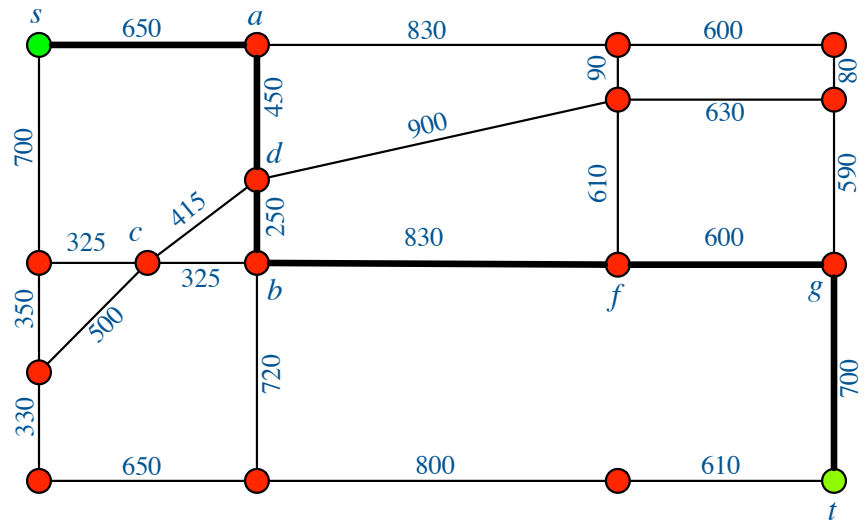


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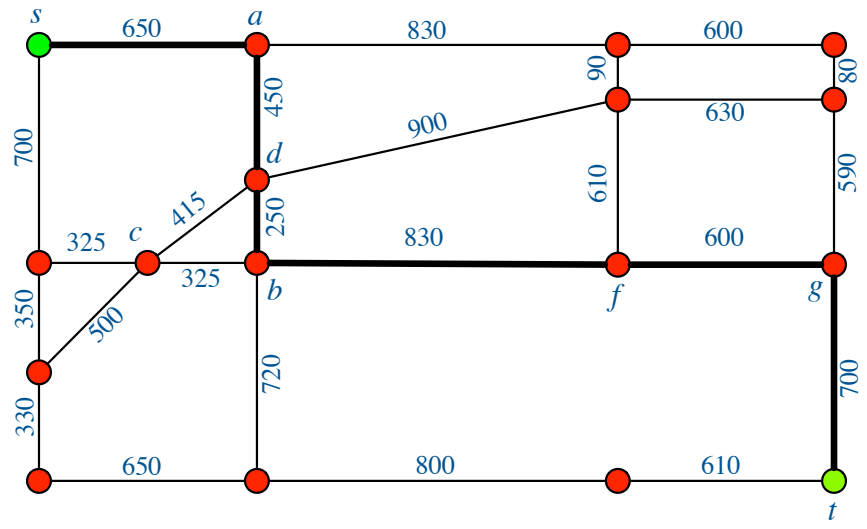
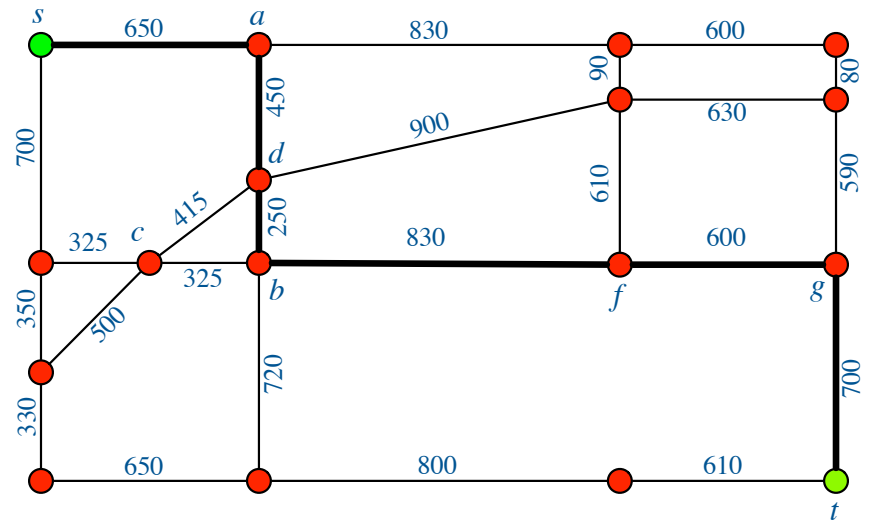



- Each edge $e \in E$ is labelled by its length $c_e \geq 0$.
- We are looking for a path connecting $s$ and $t$ of smallest total length!

# Paths




An $s,t$-path in $G = (V, E)$ is a sequence

$$v_1v_2, v_2v_3, v_3v_4, \ldots, v_{k-2}v_{k-1}, v_{k-1}v_k$$

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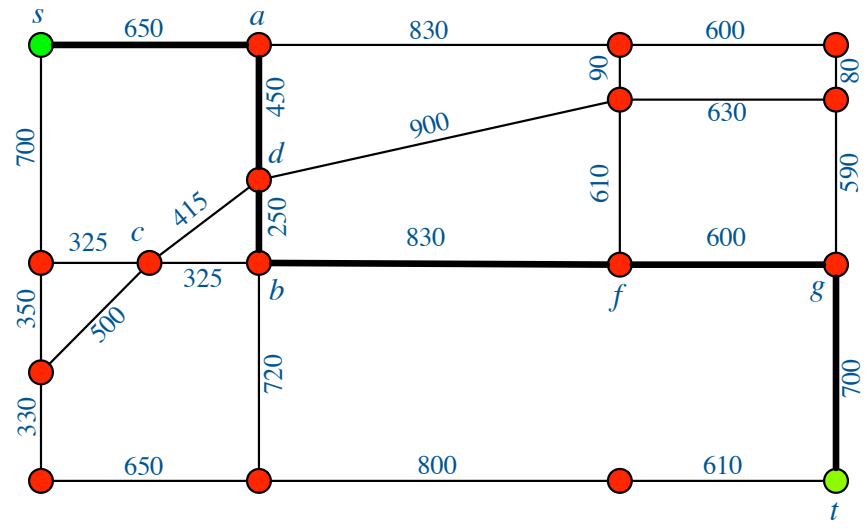


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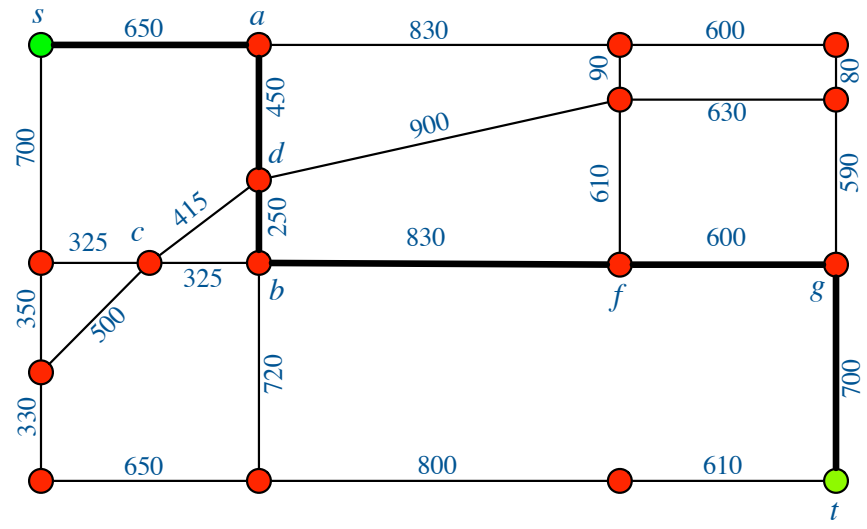


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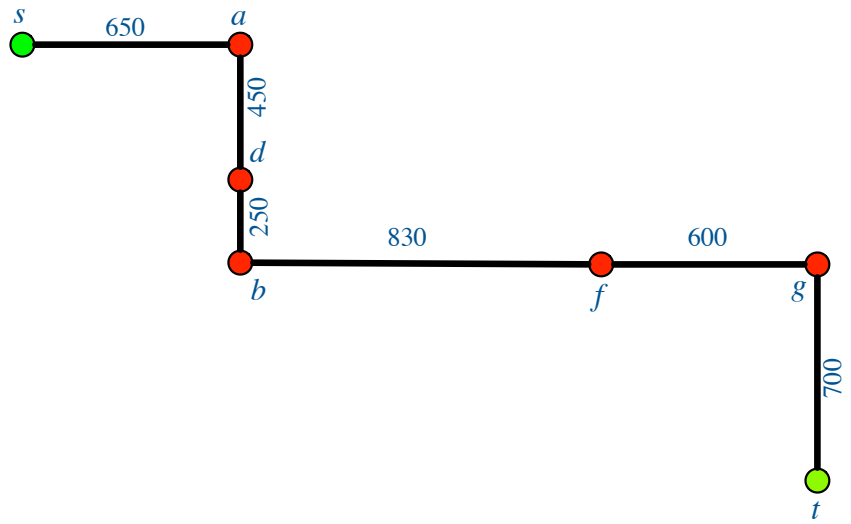


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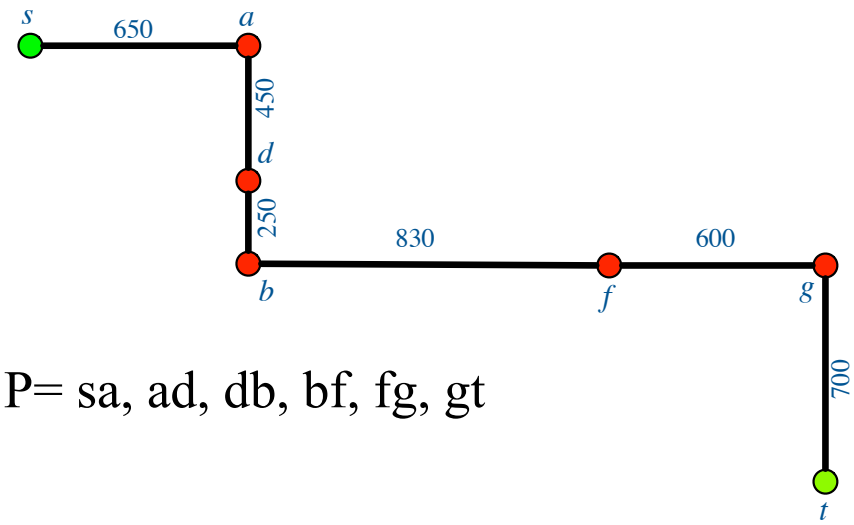


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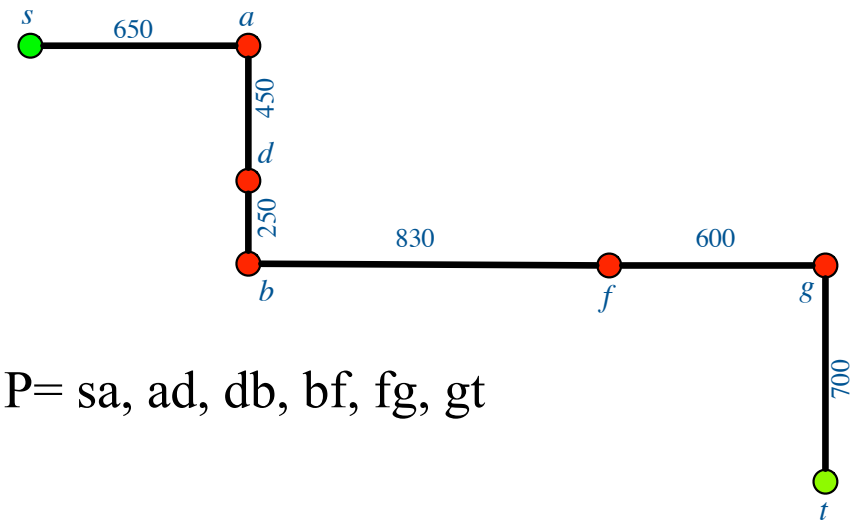


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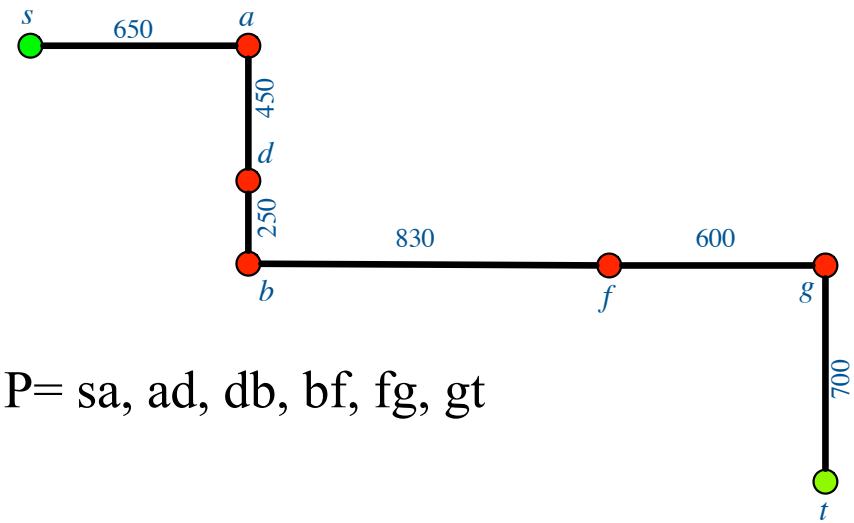


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$$\begin{aligned} c(P) &= c_{sa} + c_{ad} + c_{db} + c_{bf} + c_{fg} + c_{gt} \\ &= 650 + 490 + 250 + 830 + 600 + 700 \\ &= 3520 \end{aligned}$$

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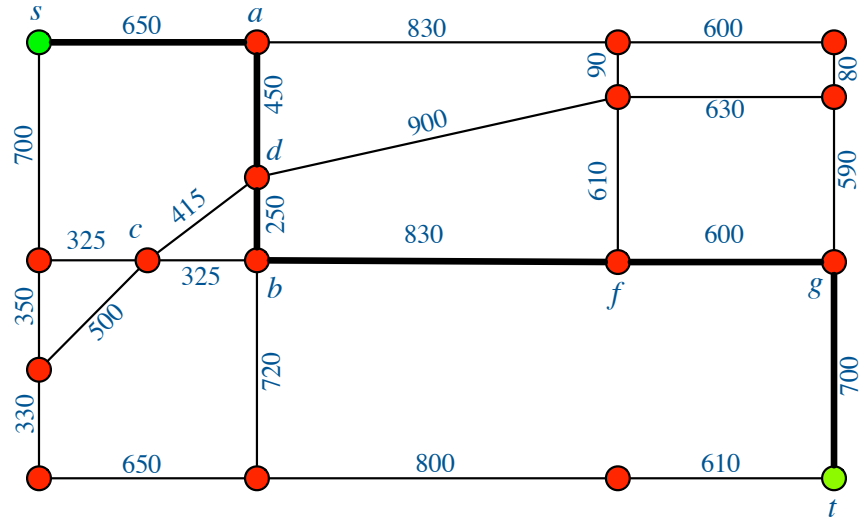


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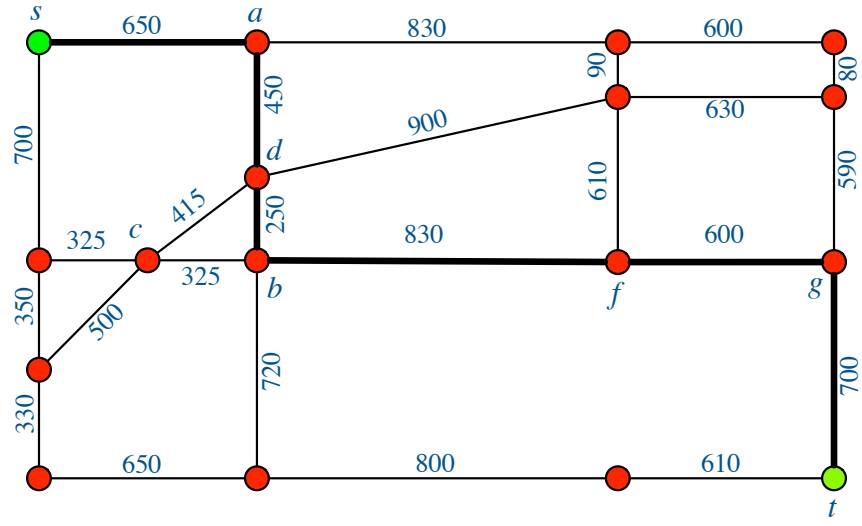


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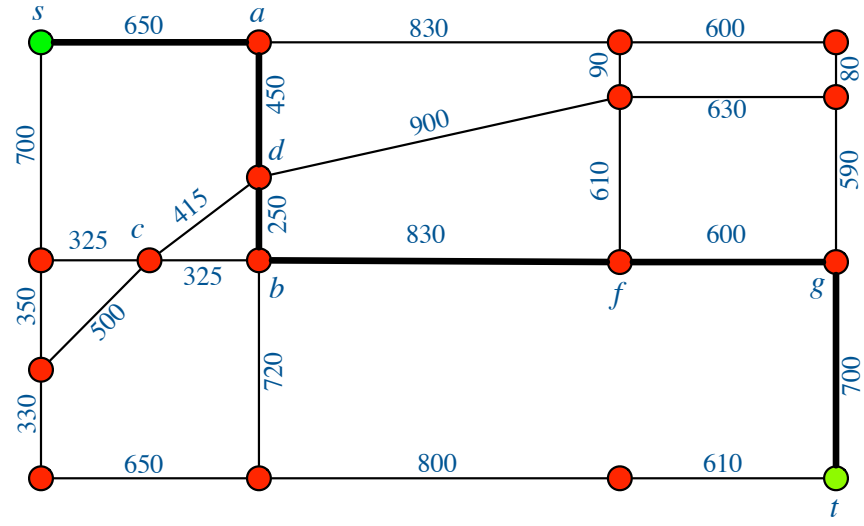


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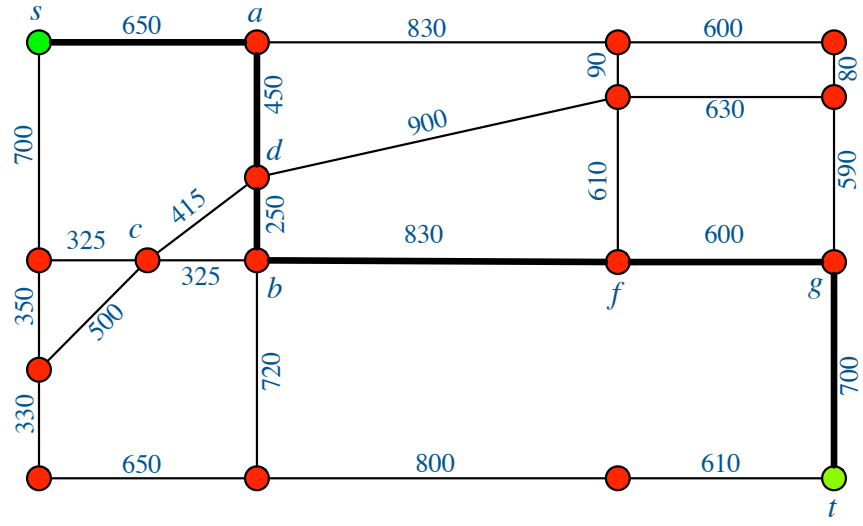


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$\longrightarrow$ Later!

# Example: Matchings

# WaterTech - Job Assignment

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$\longrightarrow$ We will rephrase this in the language of graphs

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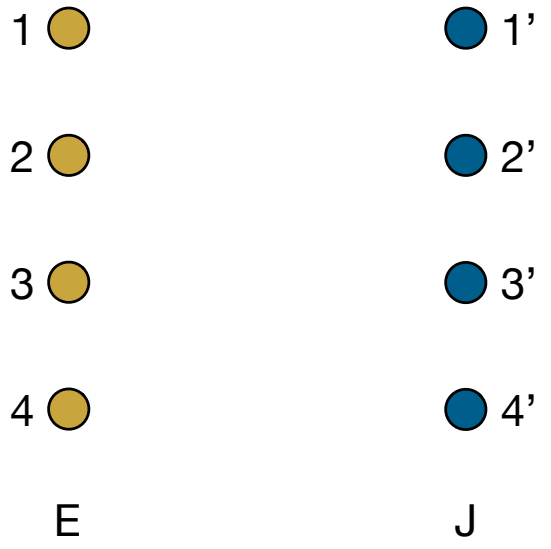

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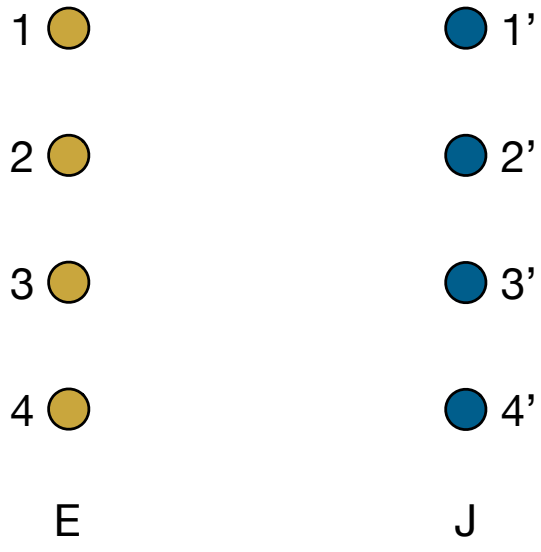


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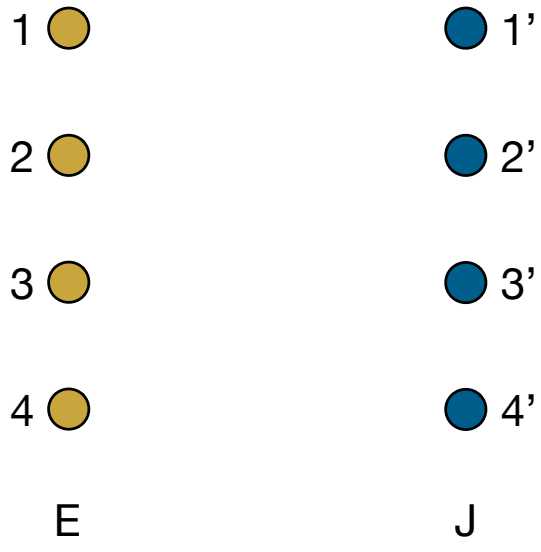
Add an edge $ij$ for $i \in E$ and $j \in J$ if employee $i$ can handle job $j$.

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| $4$ | 8 | - | 3 | - |

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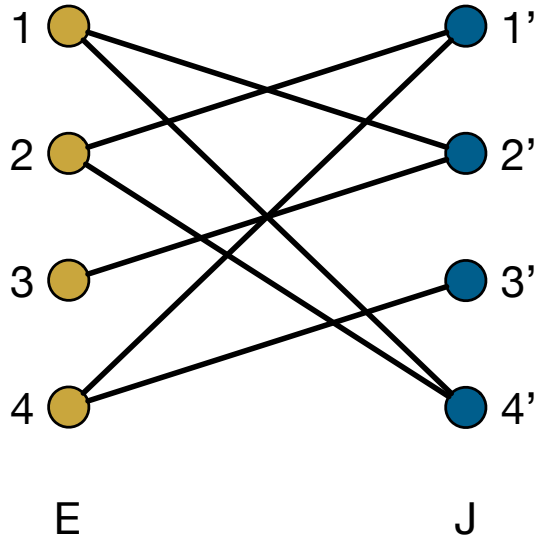
Create a graph with one vertex for each employee and job.

Add an edge $ij$ for $i \in E$ and $j \in J$ if employee $i$ can handle job $j$.




| Employees | Jobs | | | |
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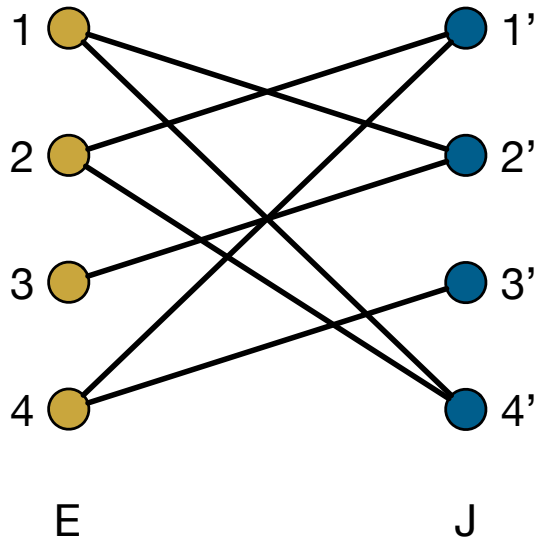


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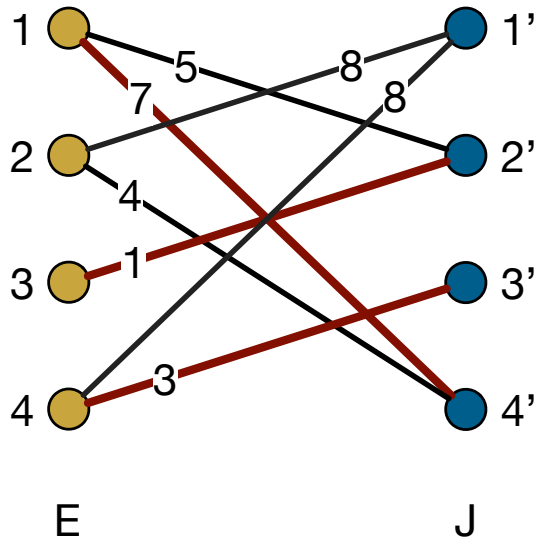


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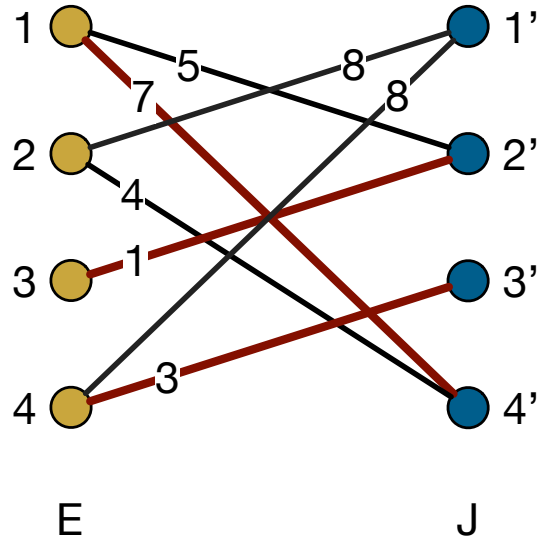

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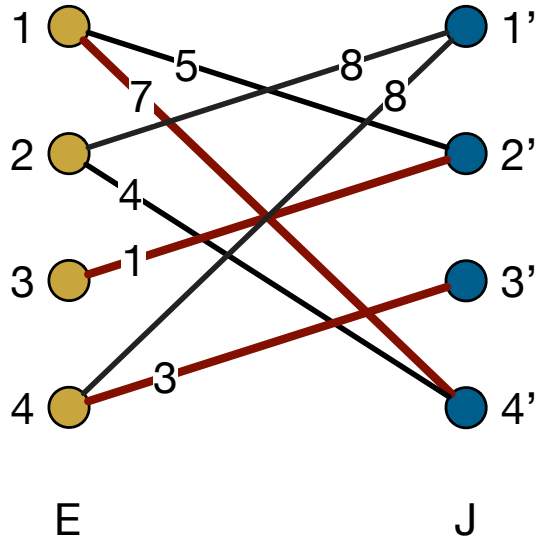

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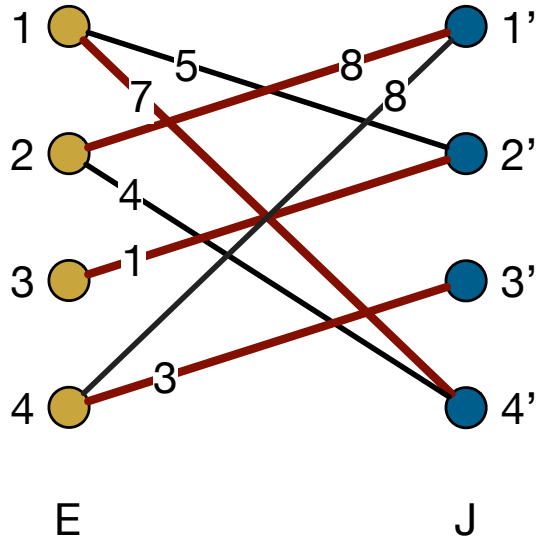


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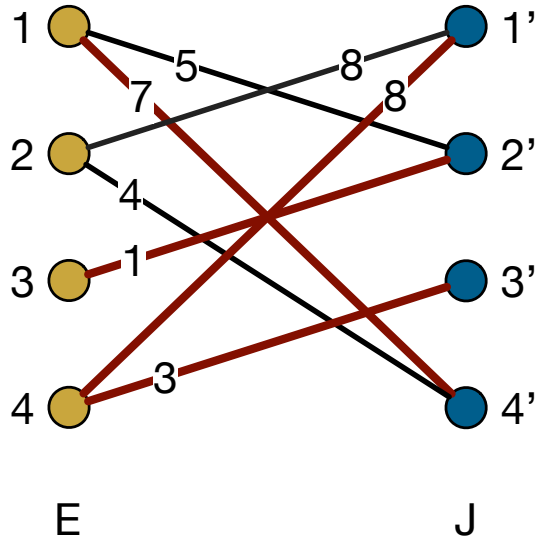


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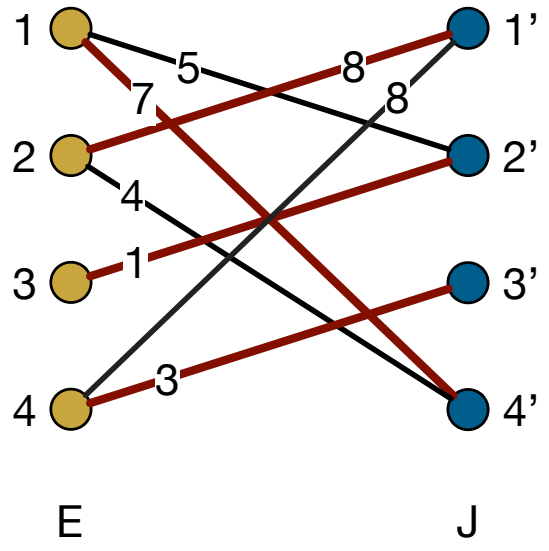

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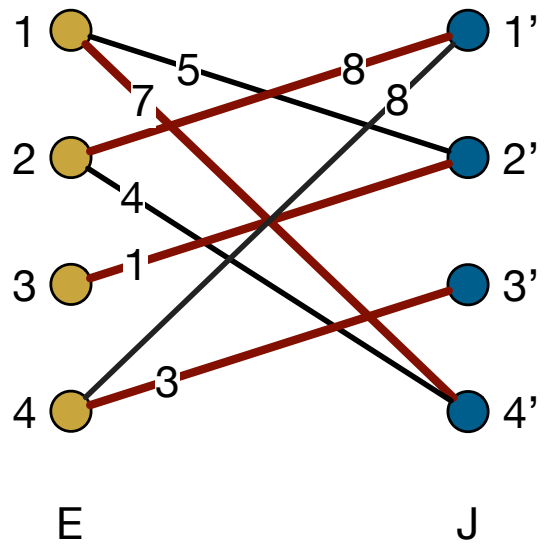

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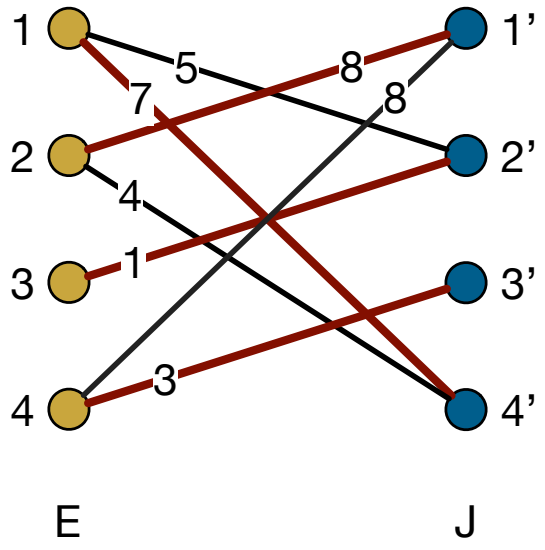


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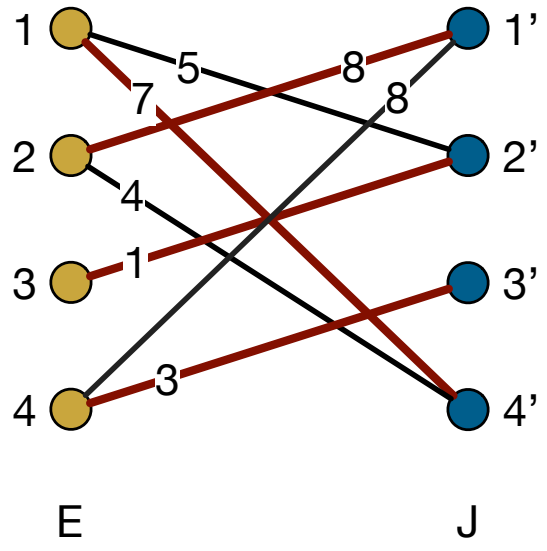


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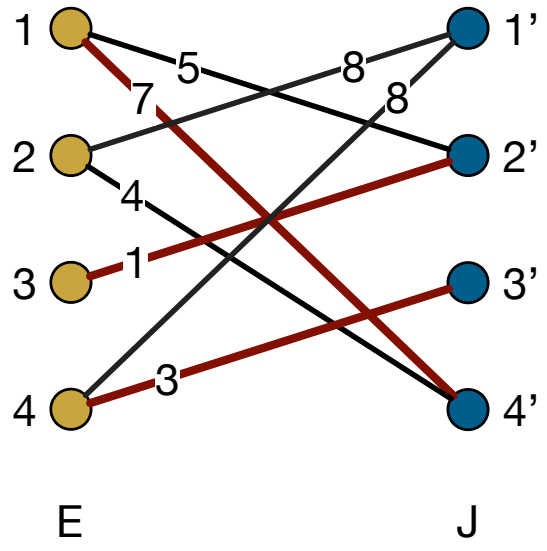


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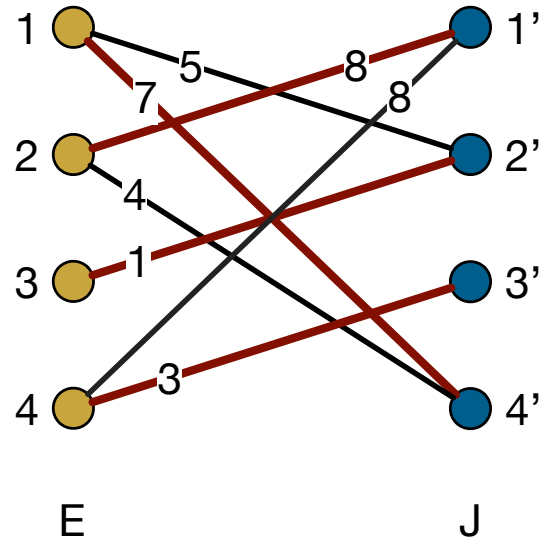

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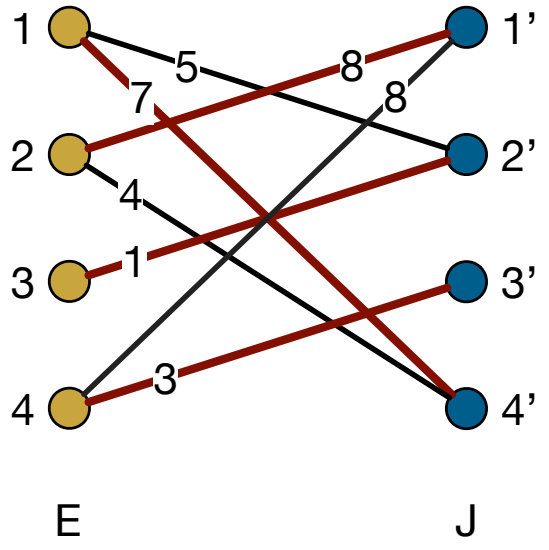


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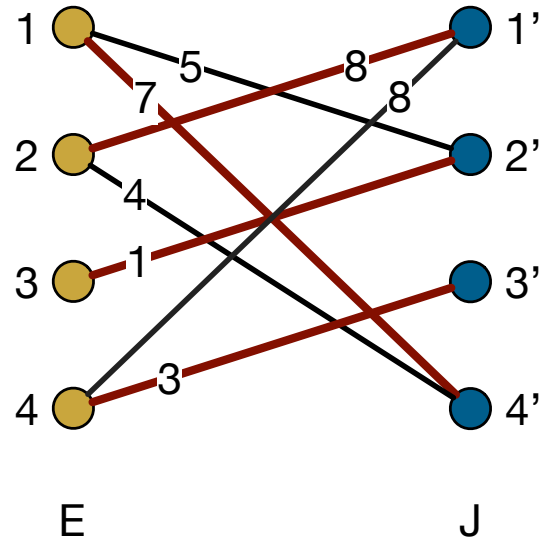


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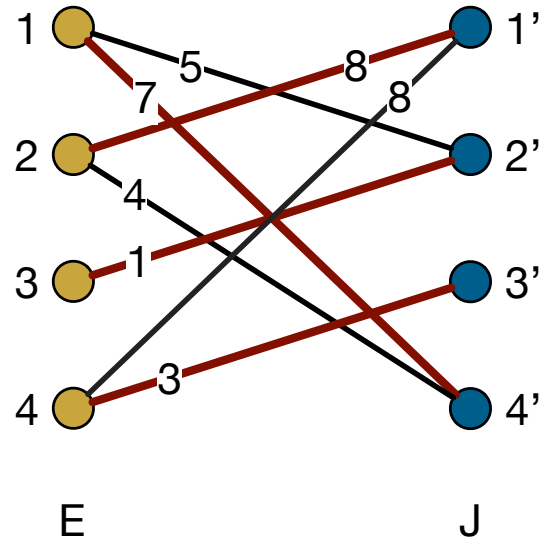


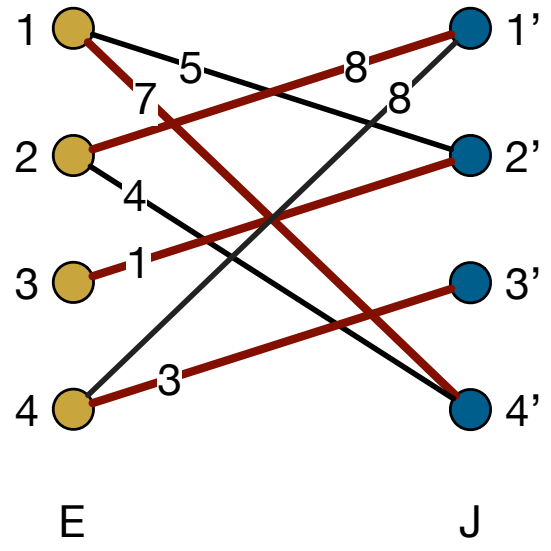

Restatement of original question:

Find a perfect matching $M$ in our graph of smallest cost.

# A Little More Notation...

Notation: Use $\delta(v)$ to denote the set of edges incident to $v$; i.e.,

$$\delta(v) = \{e \in E : e = vu \text{ for some } u \in V\}.$$



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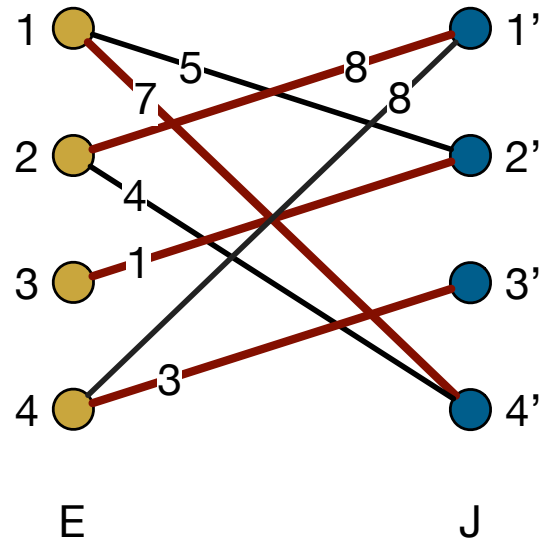


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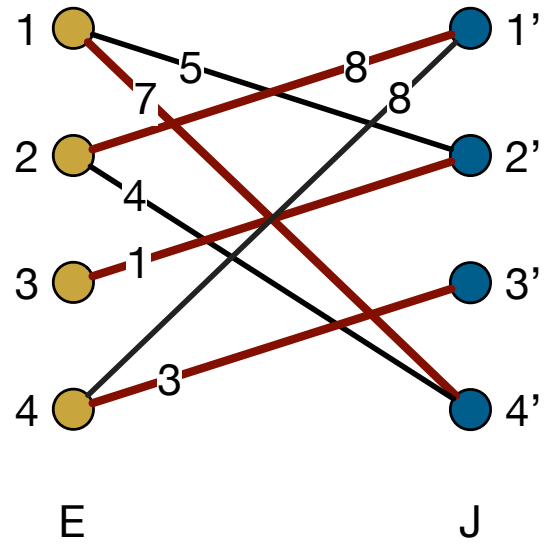


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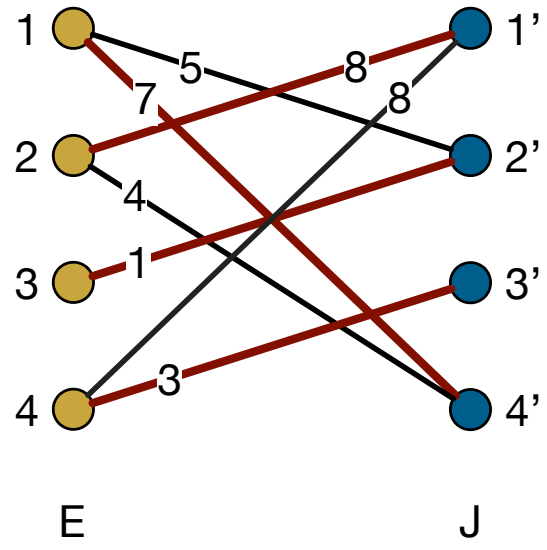


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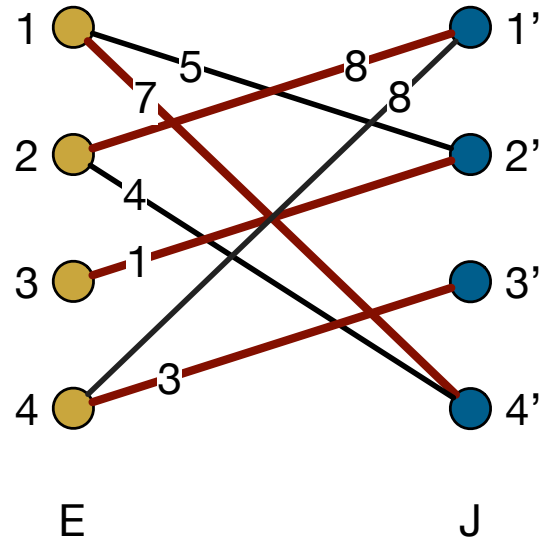

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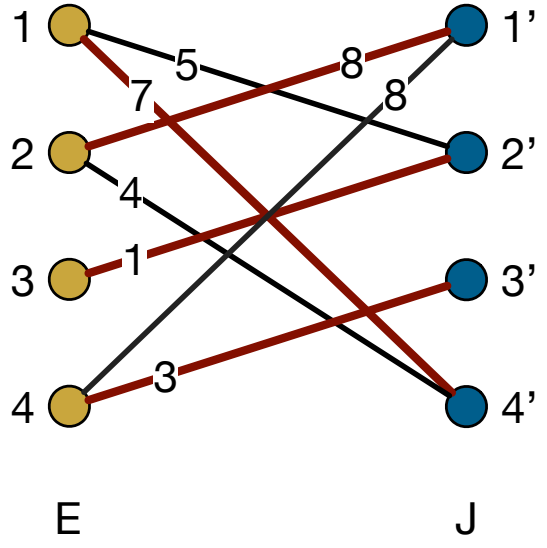

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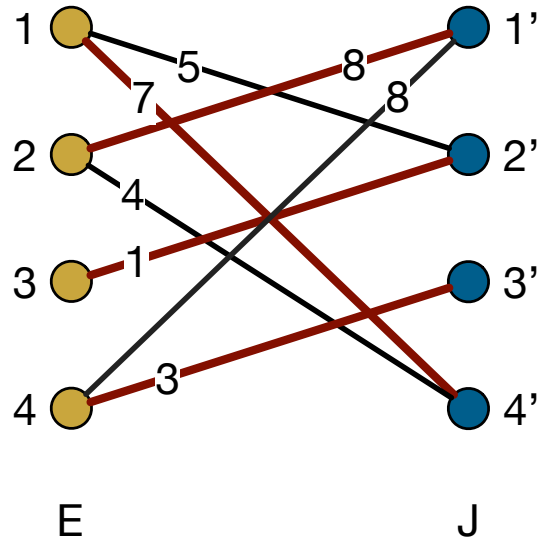

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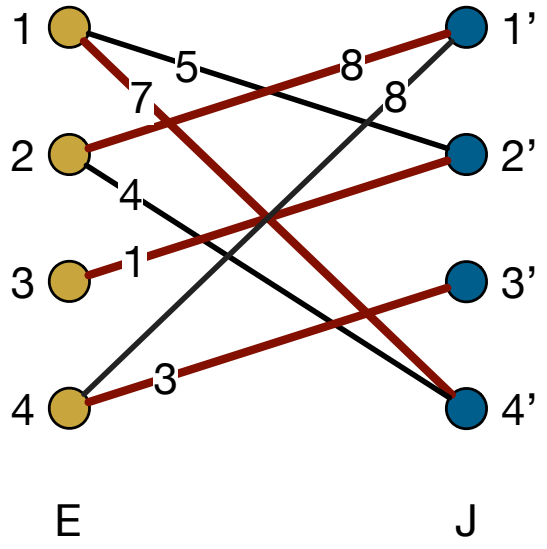


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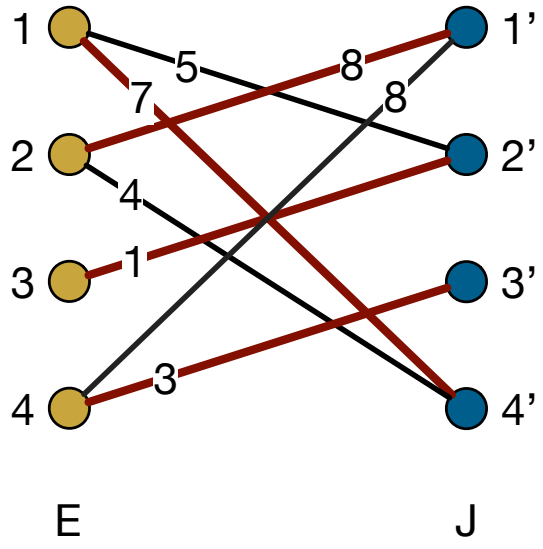


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Objective:

$$\sum(c_e x_e \,:\, e \in E)$$

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$$\min \sum(c_e x_e \ : \ e \in E)$$

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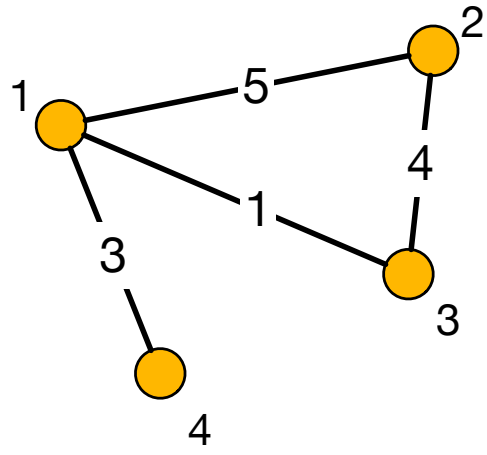

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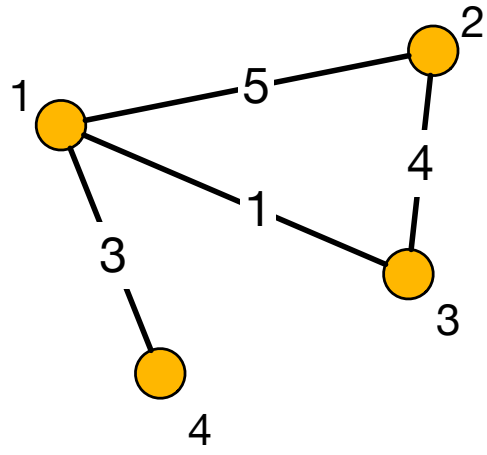


$$\min \quad (5,1,3,4)x$$

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