Medical Imaging

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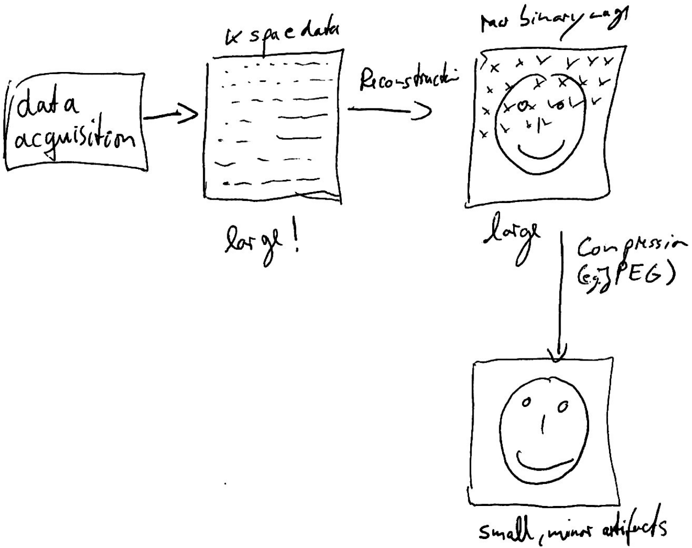
Compressed Sensing

Motivation
MRI scans are slow and usually require minutes of acquisition time.

$\Rightarrow$ Acceleration techniques wanted!

Typical Data Flow

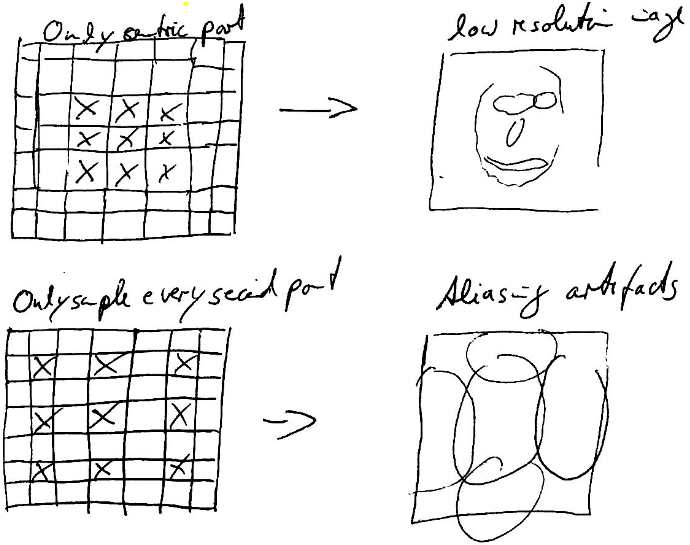


Typical Data Flow

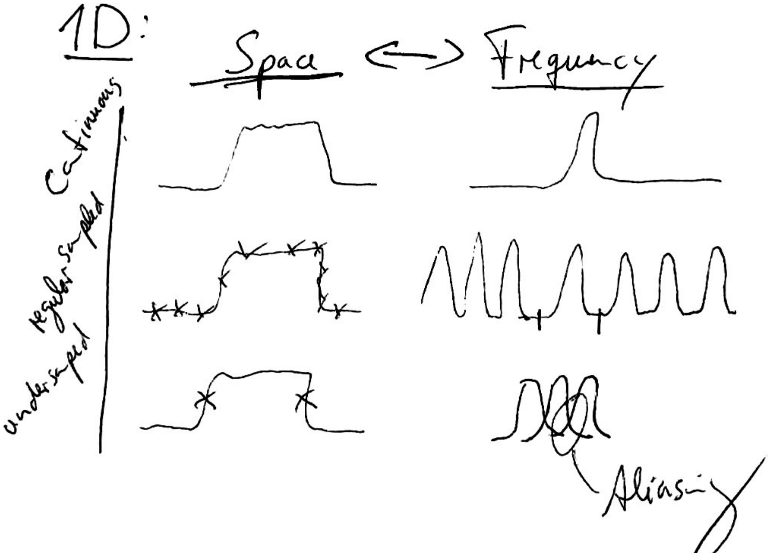
Observation
A lot of data is acquired / processed but in the end only a fraction of data is stored.

Wanted
Measure only few data and "somehow" combine the reconstruction and the compression step.

Subsampling in k-space



Subsampling in *k*-space



Nyquist Criterion

Sampling frequency has to be twice the signal bandwidth

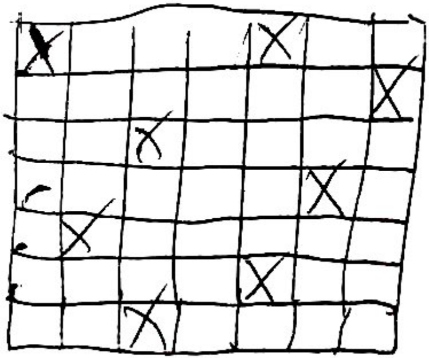
MRI: sampling in frequency space

Can Nyquist Criterion be Beaten?

- Not if the sampling is done in an equidistant way. This is always assumed when deriving the Nyquist criterion.
- If the sampling is done at random points one can beat the Nyquist criterion.
- Equidistant sampling is also named *coherent* sampling, while non-equidistant sampling is named *incoherent* sampling.

Ingredient 1 for Compressed Sensing

Incoherent sampling



Ingredient 2 for Compressed Sensing

Why is it possible to reduce the file size with JPEG?

- Most images are *compressible*.
- The second ingredient therefore is that the underlying image is compressible.

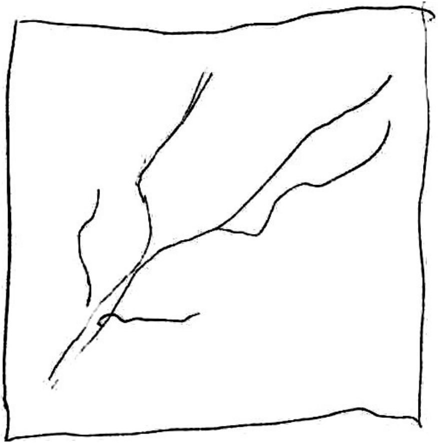
Compressibility

- Express much information with only “few coefficients”
- few coefficients: $\rightarrow$ *sparsity*

Sparsity in Image Space

Only few pixels are non-zero.

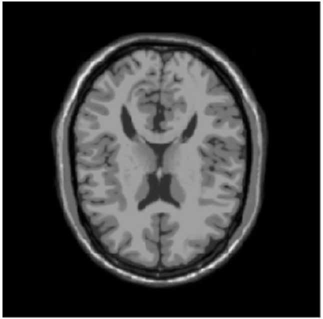
Example: angiogram



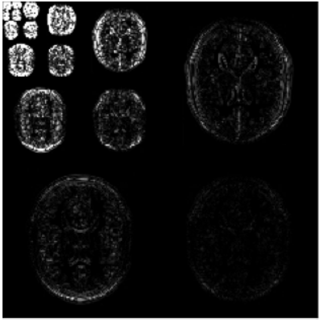
Sparsity in under Transformation

If the data is not sparse in image space one can usually apply a *sparsifying* transformation such as a Wavelet transform or a Block DCT.

Image Space



Wavelet Space



Remark
Wavelet transformation and block DCT are also used in regular compression algorithms.

Compressed Sensing Reconstruction

Imaging Equation

$$\boldsymbol{Ax} = \boldsymbol{b}$$

Subsampled Imaging Equation

$$\boldsymbol{A}_{\text{red}}\boldsymbol{x} = \boldsymbol{b}_{\text{red}}$$

$\Rightarrow$ underdetermined linear system

Compressed sensing reconstruction

We now seek for a sparse solution.

$\Rightarrow$ $\boldsymbol{x}$ should have few non-zero entries

Ansatz

$$\boldsymbol{x}_{\mathrm{CS}} = \operatorname*{argmin}_{\boldsymbol{x}} \underbrace{\|\boldsymbol{A}_{\mathrm{red}}\boldsymbol{x} - \boldsymbol{b}_{\mathrm{red}}\|_2^2}_{\text{data term}} + \underbrace{\lambda\|\boldsymbol{x}\|_0}_{\text{sparsity term}} \tag{1}$$

$$\|\boldsymbol{x}\|_0 := \text{number of non-zero elements in } \boldsymbol{x}$$

Compressed Sensing Reconstruction

However, using the L_0 norm leads to a very computationally intensive problem (NP-hard) that is unfeasible to compute in practice. Therefore one usually uses alternatively

$$\boldsymbol{x}_{\text{CS}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \underbrace{\|\boldsymbol{A}_{\text{red}}\boldsymbol{x} - \boldsymbol{b}_{\text{red}}\|_2^2}_{\text{data term}} + \underbrace{\lambda\|\boldsymbol{x}\|_1}_{\text{sparsity term}} \tag{2}$$

with

$$\|\boldsymbol{x}\|_1 := \sum_{n=1}^{N} |x_i|$$

$\Rightarrow$ Convex problem that can be efficiently solved.

Sparsity Transformation

In case that $\boldsymbol{x}$ is not sparse it is required to first apply a sparsity transformation before the L_1 norm is evaluated:

$$\boldsymbol{x}_{\mathrm{CS}} = \underset{\boldsymbol{x}}{\mathrm{argmin}} \underbrace{\|\boldsymbol{A}_{\mathrm{red}}\boldsymbol{x} - \boldsymbol{b}_{\mathrm{red}}\|_2^2}_{\text{data term}} + \underbrace{\lambda\|\boldsymbol{W}\boldsymbol{x}\|_1}_{\text{sparsity term}} \,. \tag{3}$$

Here, $\boldsymbol{W} \in \mathbb{C}^{N\times N}$ is the sparsity transformation matrix, e.g. a Wavelet transform or a block DCT. It is also possible to use a total-variation term for sparsification.