Medical Imaging

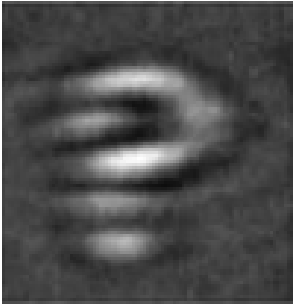
Prof. Dr. Tobias Knopp

29. November 2022

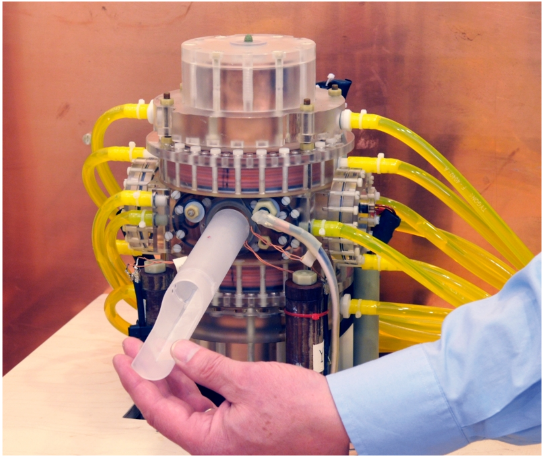
Institut für Biomedizinische Bildgebung

What is Magnetic Particle Imaging

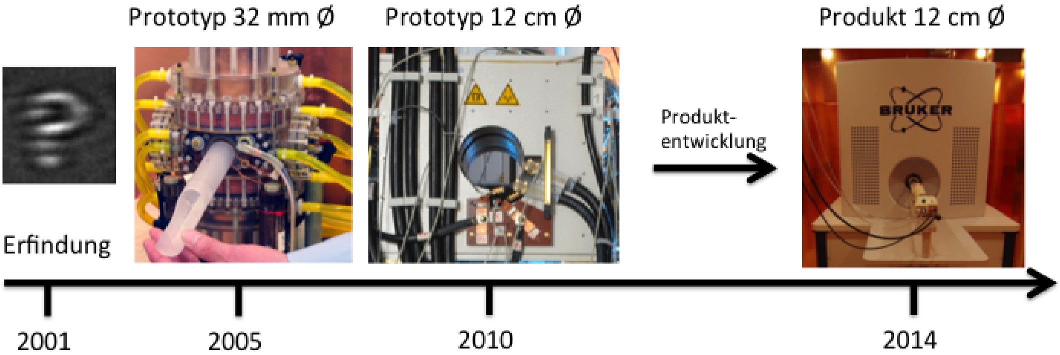
- Tomographic imaging method that allows to image super-paramagnetic nanoparticles (SPIOs)
- Invented by Bernhard Gleich in 2001 at Philips Research
- First publication: *B. Gleich and J. Weizenecker, Tomographic imaging using the nonlinear response of magnetic particles Nature. 435 30 (2005)*



First MPI Prototype



History of MPI



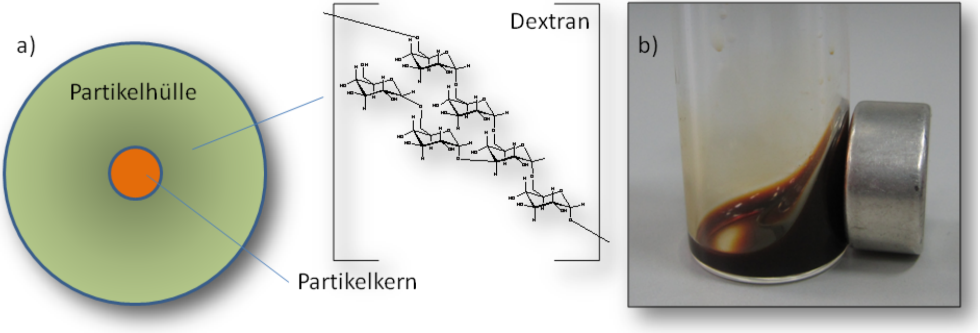
Comparison with existing Modalities

Tabelle 1: Quantitative comparison of different imaging modalities.

	CT	MRI	PET	SPECT	**MPI**
spatial resolution	0.5 mm	1 mm	4 mm	10 mm	1–3 mm
acquisition time	1 s	1 s – 1 h	1 min	1 min	< 0.1 s
sensitivity	medium	medium	very high	very high	high
quantifiability	yes	no	yes	yes	yes
harmfulness	X-ray	heating	β/γ radiation	γ radiation	heating

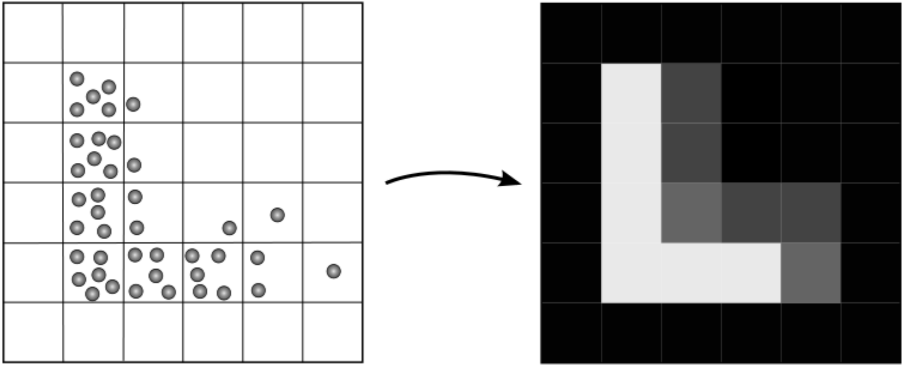
Magnetic Nanoparticles

- Particles consist of an iron-oxide core and a hull that prevents agglomeration and particle-particle interaction



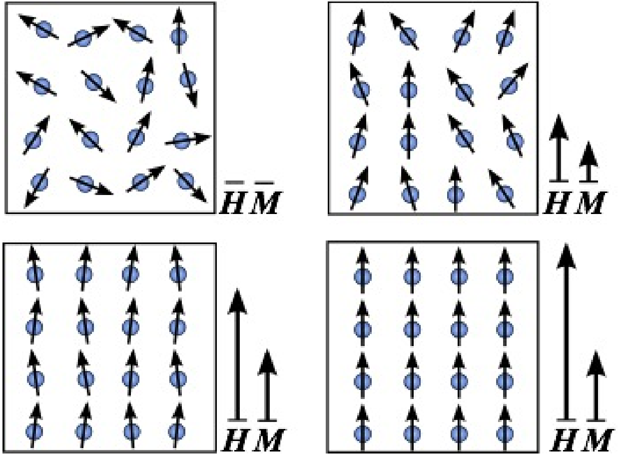
What is Magnetic Particle Imaging

- MPI images the spatially dependent concentration of SPIOs
- Concentration: Particles per Voxel



Saturation Effect

- Particles align with external magnetic field $\boldsymbol{H}$
- Saturation when all particles are aligned



Partice Magnetization

$$\boldsymbol{M} := \frac{1}{\Delta V} \sum_{j=0}^{N^{\mathrm{P}}-1} \boldsymbol{m}_j \tag{1}$$

where $\boldsymbol{m}_j$ are the magnetic moments within a voxel.

Under equilibrium assumptions $\boldsymbol{M}$ can be expressed as

$$\boldsymbol{M}(\boldsymbol{H}) = M(H)\boldsymbol{e}_{\boldsymbol{H}}, \tag{2}$$

where $\boldsymbol{e}_{\boldsymbol{H}}$ is the direction of the magnetic field and

$$M(H) = c\, m\mathcal{L}\left(\beta H\right) \tag{3}$$

is the length of the magnetization vector in dependence of the strength of the magnetic field $H := \|\boldsymbol{H}\|_2$.

Particle Magnetization

$M(H)$ depends on the Langevin function

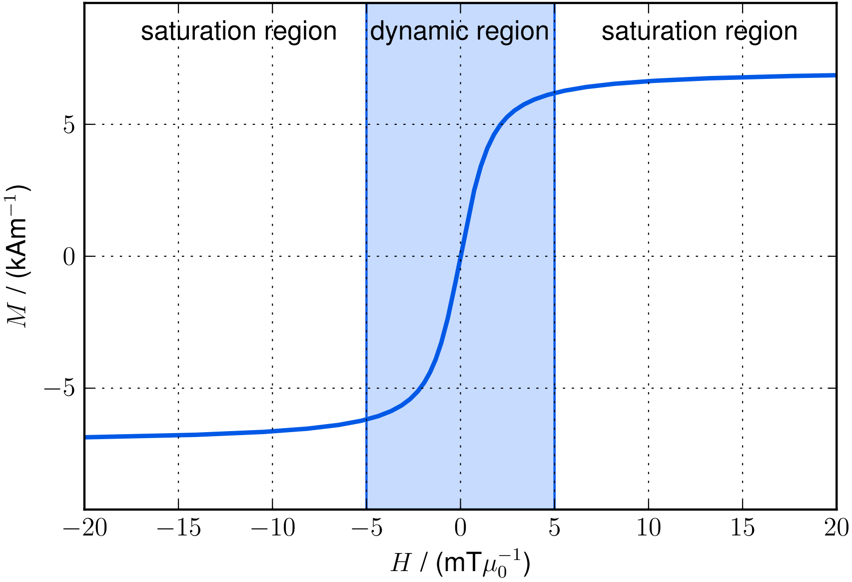
$$\mathcal{L}(\xi) := \begin{cases} \coth(\xi) - \frac{1}{\xi} & \xi \neq 0 \\ 0 & \xi = 0 \end{cases} \tag{4}$$

and the scaling factor

$$\beta := \frac{\mu_0 m}{k_\mathrm{B} T^\mathrm{P}}. \tag{5}$$

μ_0 is the permeability of free space, k_B is the Boltzmann constant, T^P is the particle temperature, and $m = V_\mathrm{core} M^\mathrm{S}_\mathrm{core}$ is the magnetic moment of a single particle. The latter is determined by the saturation magnetization of the material $M^\mathrm{S}_\mathrm{core}$ of which the particle core is made (usually magnetite) and the particle core volume $V_\mathrm{core} = \frac{1}{6}\pi D^3_\mathrm{core}$ derived from the core diameter D_core.

Particle Magnetization



Signal and Spatial Encoding

Any tomographic imaging method needs two ingredients:

- Signal Encoding
- Spatial Encoding

Signal Encoding Signal encoding describes the process that the underlying tomographic image generates some kind of signal.

Spatial Encoding Spatial encoding describes, how the spatial position of a voxel can be encoded into the signal. Usually this means to create a spatial dependency of the signal.

Signal and Spatial Encoding

Remark Signal and spatial encoding are in the end happening simultanuously. The do, however, help understanding the imaging methods conceptually.

Example In computed tomography, the signal is encoded by passing an X-ray through the object. This also partly does spatial encoding in one direction of the imaging plane.

Full spatial encoding is achieved by rotation of the gantry. This leads to the station that the signal response of a delta peak in image space yields a different fingerprint in the raw data signal, i.e. no two positions yield the same sinogram.

Signal Generation

Signal generation in MPI is done by cyclic excitation of the magnetic nanoparticles using dynamic magnetic fields. To illustrate the signal generation consider a homogeneous sinusoidal drive field

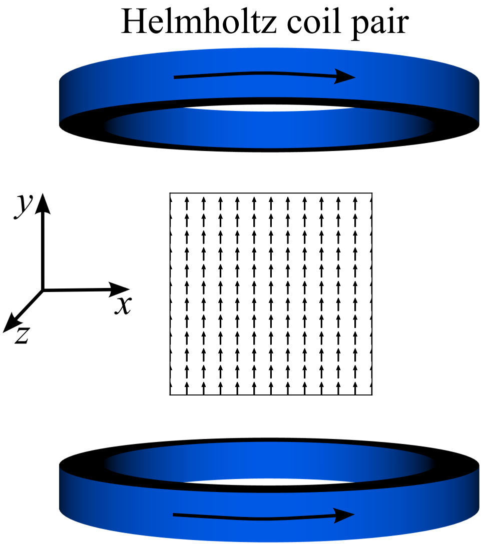
$$\boldsymbol{H}^{\mathrm{D}}(t) = -A\cos(2\pi f t)\boldsymbol{e}^{\boldsymbol{H}}, \tag{6}$$

with field amplitude A, frequency f, and field direction $\boldsymbol{e}^{\boldsymbol{H}}$. If an ensemble of Langevin particles is excited by this field it generates the signal

$$\boldsymbol{M}(t) = \boldsymbol{M}(\boldsymbol{H}^{\mathrm{D}}(t)), \tag{7}$$

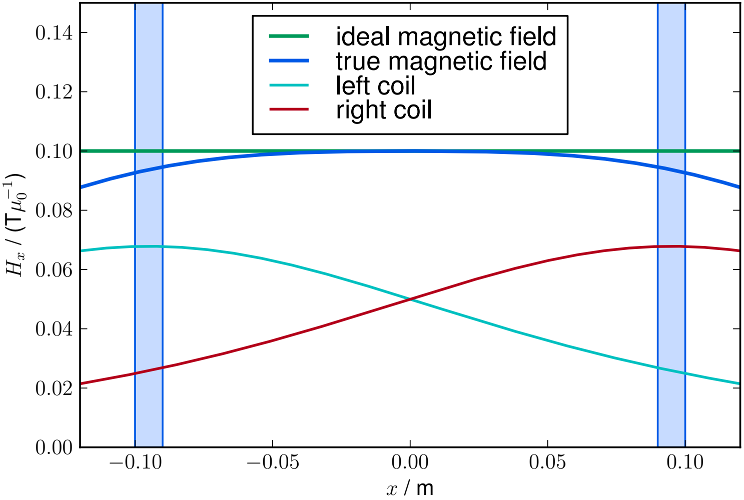
Signal Encoding

- A homogeneous field can be generated by two coils with currents flowing in the same direction.

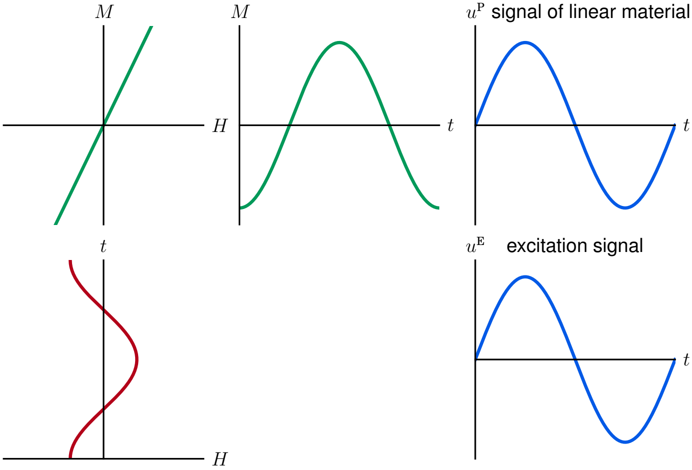


Signal Encoding

Magnetic field is not perfectly homogenous

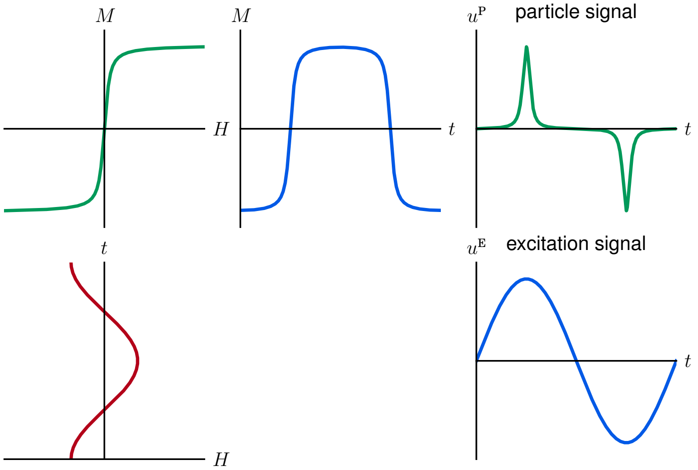


Signal Encoding



Linear material $\rightarrow$ excitation signal and particle signal cannot be distinguished.

Signal Encoding



Non-linear material $\rightarrow$ excitation signal and particle signal **can** be distinguished.

MPI Imaging Equation – Frequency Space

The voltage signal $u(t)$ is periodic and allows us to expand the voltage signal $u(t)$ into a Fourier series:

$$u(t) = \sum_{k=-\infty}^{\infty} \hat{u}_k \mathrm{e}^{2\pi \mathrm{i} k t / T}$$

and the spectrum consists of discrete lines at multiples of the frequency $f = 1/T$, which is also called the fundamental or base frequency. These multiples

$$f_k = kf, k \in \mathbb{Z} \tag{8}$$

are usually called harmonic frequencies or just harmonics. The Fourier coefficients can be computed by

$$\hat{u}_k = \frac{1}{T} \int_0^T u(t) \mathrm{e}^{-2\pi \mathrm{i} k f t} \mathrm{d}t, \quad k \in \mathbb{Z}. \tag{9}$$

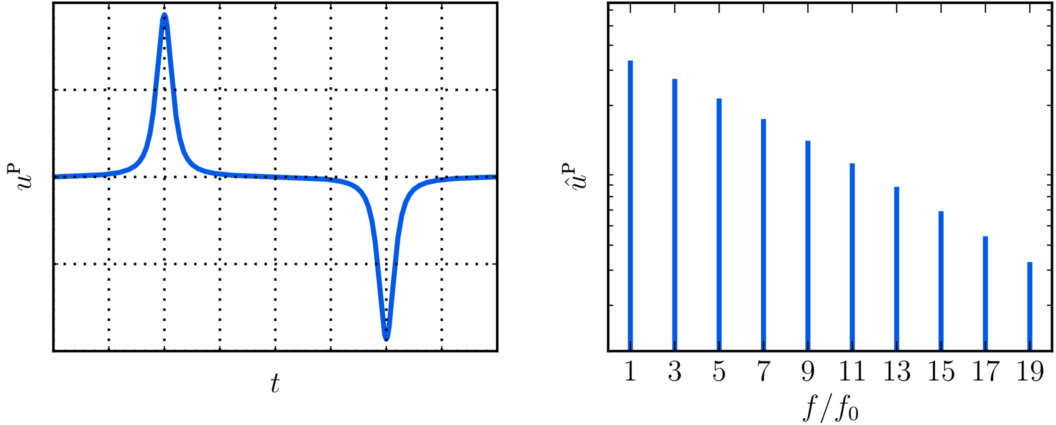
MPI Imaging Equation – Frequency Space

As the induced voltage is real, the Fourier coefficients obey the relation

$$\begin{aligned} \hat{u}_k &= \frac{1}{T}\int_0^T u(t)\mathrm{e}^{-2\pi \mathrm{i} kft}\mathrm{d}t \\ &= \frac{1}{T}\int_0^T \left(u(t)\mathrm{e}^{2\pi \mathrm{i} kft}\right)^* \mathrm{d}t \\ &= \left(\hat{u}_{-k}\right)^* . \end{aligned} \tag{10}$$

Therefore, one usually neglects the negative frequencies in MPI as they do not carry any additional information.

Signal Encoding



Signal Encoding

The generation of higher harmonics for a non-linear magnetization curve can be mathematically described by expanding the Langevin function into a Taylor series

$$\mathcal{L}(\xi) = \frac{1}{3}\xi - \frac{1}{45}\xi^3 + \frac{2}{954}\xi^5 - \frac{1}{4725}\xi^7 + \dots\ . \tag{11}$$

If one considers the particle magnetization M, one can see that the argument $\frac{\mu_0 Hm}{k_B T^P}$ is applied to the Langevin function. For a sinusoidal field excitation $H(t) = -A\cos(2\pi ft)$, the dynamic part of the particle magnetization can be written as

$$\mathcal{L}(\tilde{\xi}\cos(2\pi ft)) = \frac{\tilde{\xi}}{3}\cos(2\pi ft) - \frac{\tilde{\xi}^3}{45}\cos^3(2\pi ft) + \dots\ , \tag{12}$$

where $\tilde{\xi} = -\frac{\mu_0 Am}{k_B T^P}$.

Signal Encoding

Using the trigonometric formula

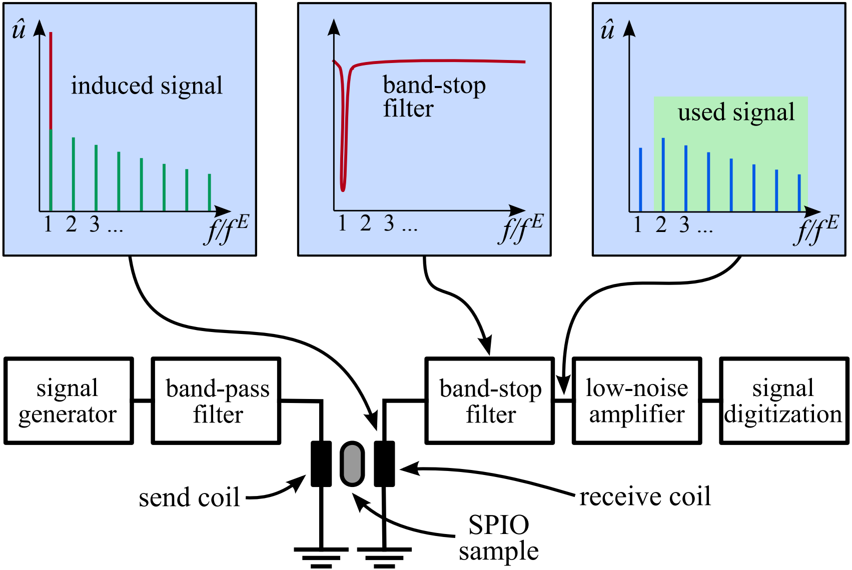
$$\cos^3(x) = \frac{1}{4}\left(3\cos(x) + \cos(3x)\right), \tag{13}$$

one obtains

$$\begin{aligned}\mathcal{L}(\tilde{\xi}\cos(2\pi ft)) &= \frac{\tilde{\xi}}{3}\cos(2\pi ft) - \frac{\tilde{\xi}^3}{60}\cos(2\pi ft) + \frac{\tilde{\xi}^3}{180}\cos(2\pi(3f)t) + \ldots \\ &= \frac{20\tilde{\xi} - \tilde{\xi}^3}{60}\cos(2\pi ft) + \frac{\tilde{\xi}^3}{180}\cos(2\pi(3f)t) + \ldots\,.\end{aligned} \tag{14}$$

Hence, the third harmonic, which corresponds to the frequency $3f$ is present in the spectrum of the induced voltage for a sinusoidal excitation. By including higher order terms $\cos^5$, $\cos^7$, $\ldots$, one can verify that all odd harmonics are present in the signal spectrum. The even harmonics are missing, as all even derivatives of the Langevin function have a zero-crossing at the point $\xi = 0$, at which the Taylor series is expanded.

Analog Signal Chain



Spatial Encoding

Recall at this point that $\boldsymbol{H}^{\mathrm{D}}$ is homogeneous and thus all particles in space behave the same.

What we do next is to superimpose a second magentic field $\boldsymbol{H}^{\mathrm{S}}(\boldsymbol{r})$, which is static but spatially dependent:

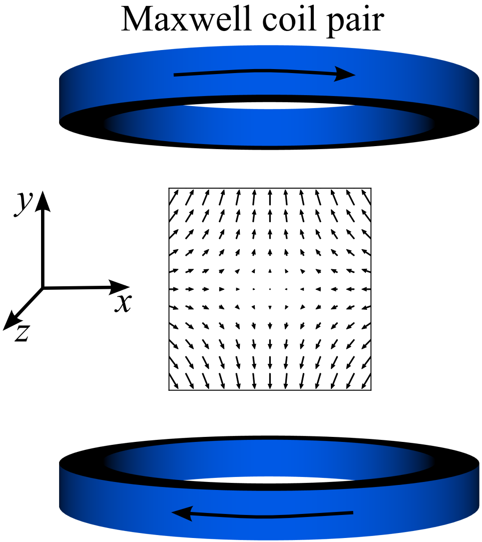
$$\boldsymbol{H}_{\mathrm{S}}(\boldsymbol{r}) = \begin{pmatrix} G_x & 0 & 0 \\ 0 & G_y & 0 \\ 0 & 0 & G_z \end{pmatrix} \boldsymbol{r} \tag{15}$$

The effective excitation signal

$$\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{H}^{\mathrm{D}}(t) + \boldsymbol{H}^{\mathrm{S}}(\boldsymbol{r}) \tag{16}$$

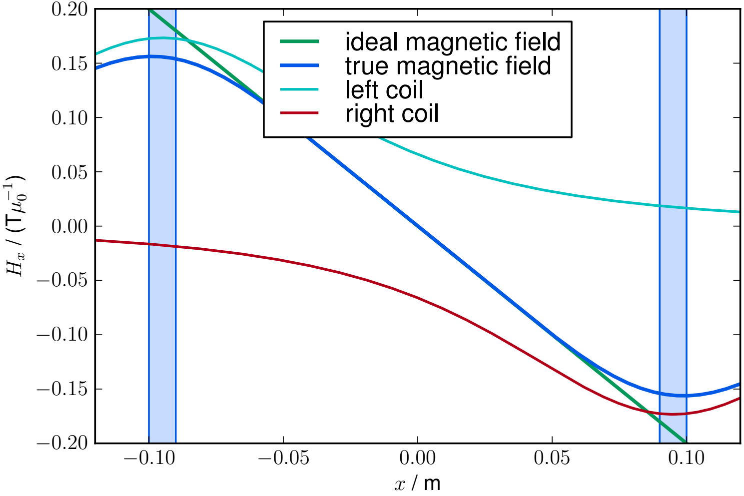
is thus unique at each spatial position.

- The gradient field $\boldsymbol{H}^{\mathrm{S}}$ has a field-free point in the center.
- The field increases in all directions in space.
- It can be generated using two coils and current flowing in opposing directions.

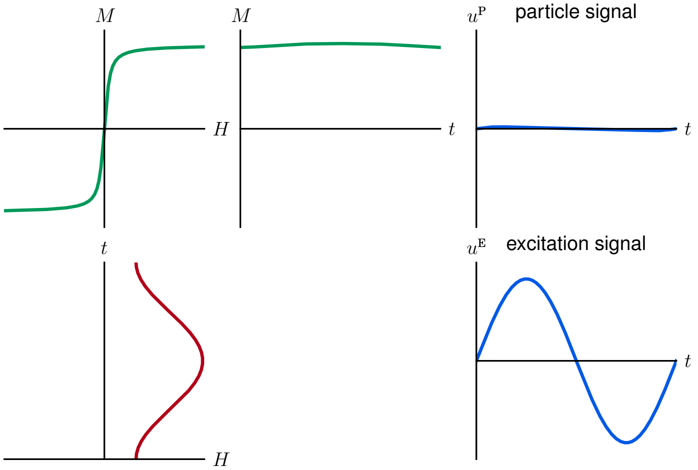


Signal Encoding

Magnetic field is not perfectly linear



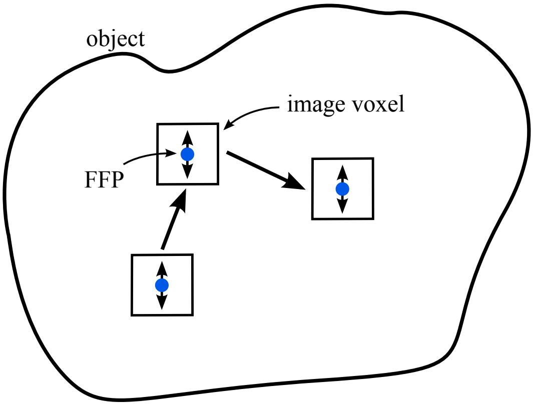
Spatial Encoding



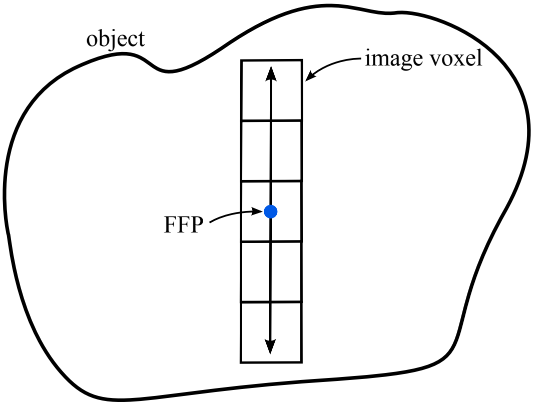
Large offset supresses signal.

Signal Encoding

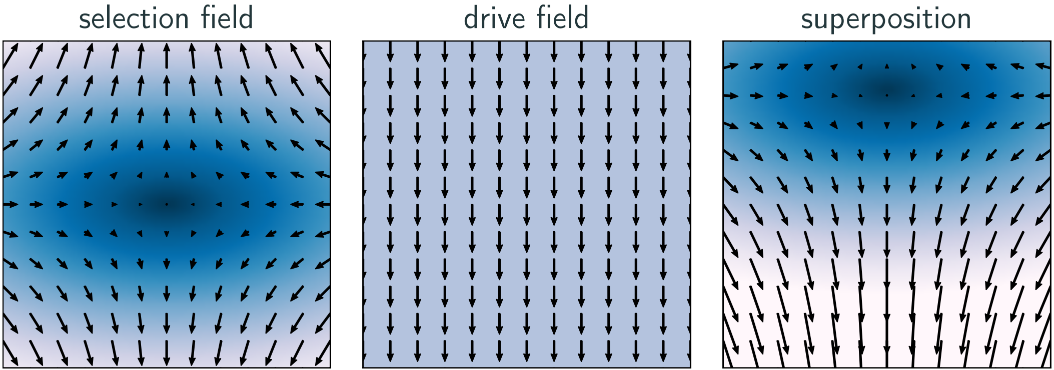
Single-voxel imaging – not very effective



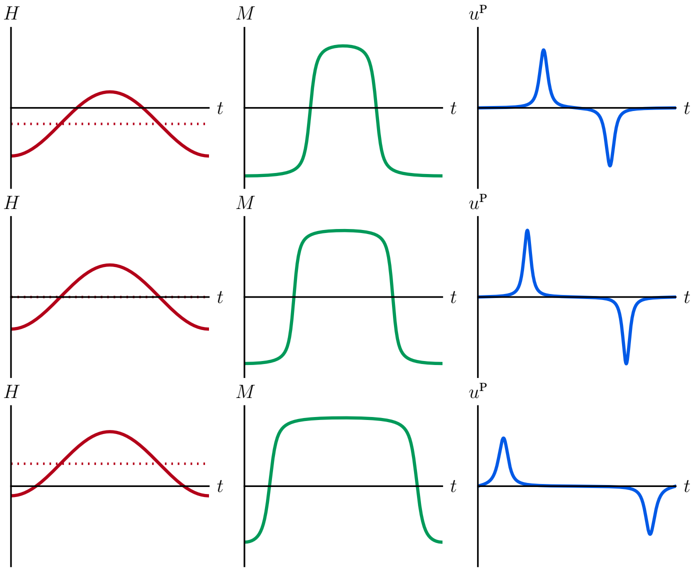
Line imaging – much more effictive



FFP Shift



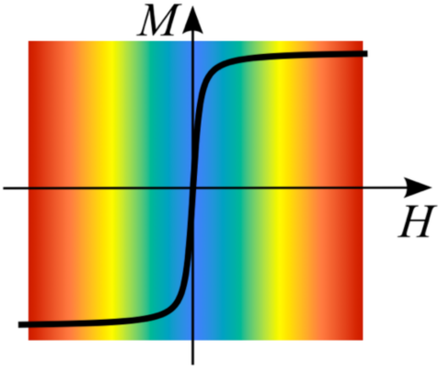
Spatial Encoding



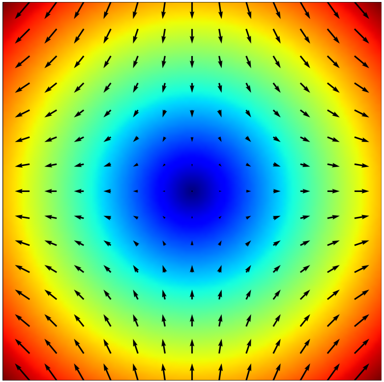
- Each point in space generates a different signal
- Basically the signal peak is shifted

Spatial Encoding

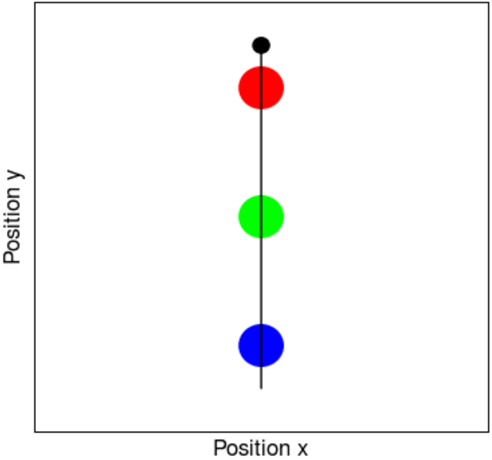
Particle Magnetization

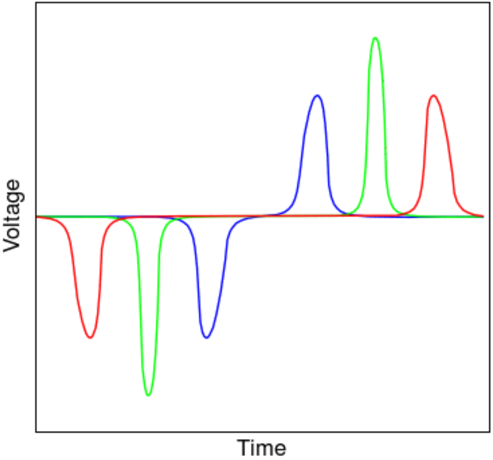


Gradient Field

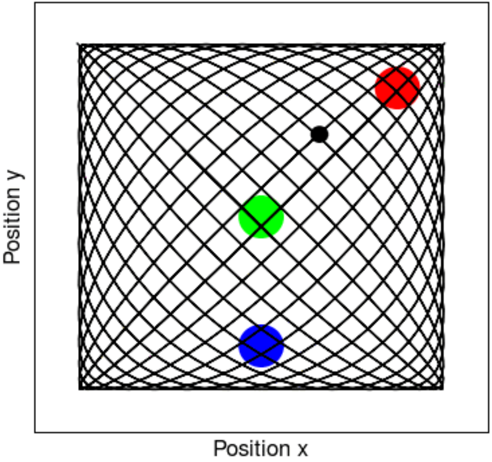


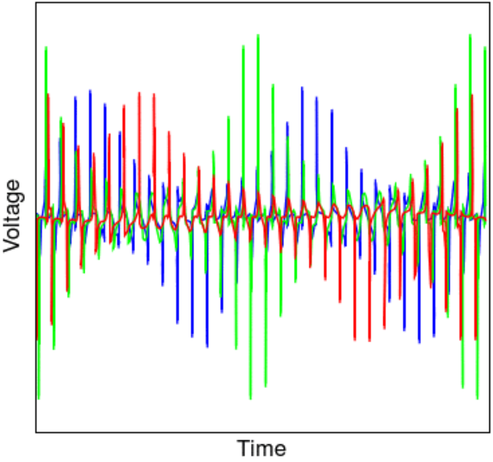
1D Imaging Sequence



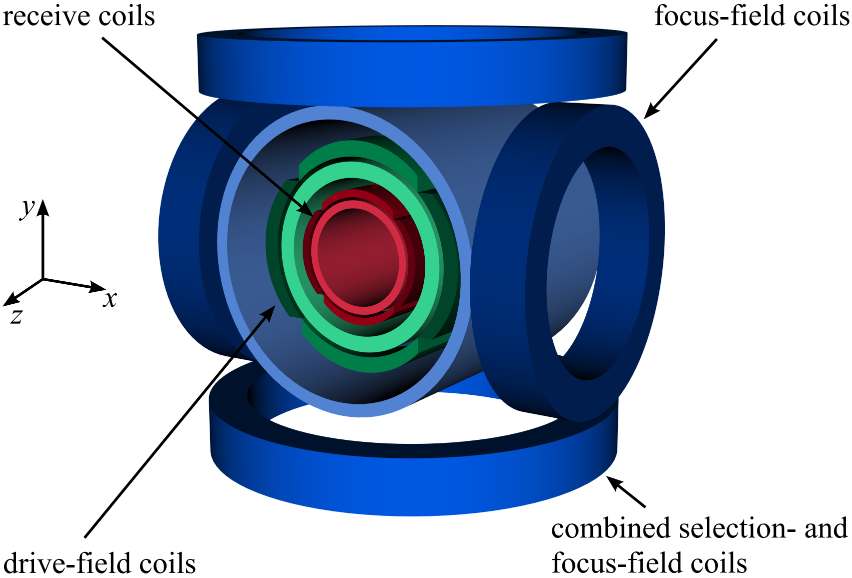


2D Imaging Sequence





MPI Coil Setup



MPI Imaging Equation – Time Space

Theorem
The relation between the particle distribution c and the voltage u induced in a receive coil with sensitivity $\boldsymbol{p}$ is linear and can be expressed as

$$u(t) = \int_{\mathbb{R}^3} s(\boldsymbol{r}, t) c(\boldsymbol{r}) d^3 r, \tag{17}$$

where

$$s(\boldsymbol{r}, t) = -\mu_0 \boldsymbol{p}(\boldsymbol{r}) \cdot \frac{\partial \overline{\boldsymbol{m}}(\boldsymbol{r}, t)}{\partial t}. \tag{18}$$

denotes the system function in time space.

MPI Imaging Equation – Frequency Space

Theorem
The relation between the particle distribution c and the frequency components of the induced voltage $\hat{u}_k$ is linear and can be expressed as

$$\hat{u}_k = \int_{\mathbb{R}^3} \hat{s}_k(\boldsymbol{r}) c(\boldsymbol{r}) d^3 r. \tag{19}$$

where

$$\hat{s}_k(\boldsymbol{r}) = -\frac{\mu_0}{T} \int_0^T \boldsymbol{p}(\boldsymbol{r}) \cdot \frac{\partial \overline{\boldsymbol{m}}(\boldsymbol{r}, t)}{\partial t} e^{-2\pi \mathrm{i} k t / T} dt \tag{20}$$

denotes the system function in frequency space.

Discrete MPI Imaging Equation

Sampling of time and space leads to

Discrete Setting

$$\hat{u}_k = \sum_{n=1}^{N} s_{k,n} c_n \quad \Leftrightarrow \boldsymbol{u} = \boldsymbol{S}\boldsymbol{c}$$

where

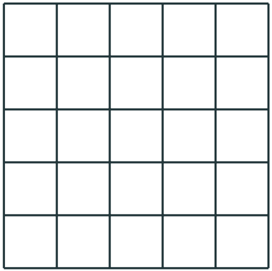
$$\begin{aligned} k &\in I_K, \\ I_K &= \{1, \dots, K\}, \\ \boldsymbol{u} &= \left(u_k\right)_{k \in I_K}, \\ \boldsymbol{c} &= \left(c_n\right)_{n \in I_N}, \\ \boldsymbol{S} &= \left(s_{k,n}\right)_{k \in I_K; n \in I_N} \end{aligned}$$

Remark

- The sampling positions $\boldsymbol{r}_n$, $n \in I_N$ represent a 2D / 3D grid. E.g.

$$\boldsymbol{r}_n = \boldsymbol{r}_{n_x,n_y,n_z}$$

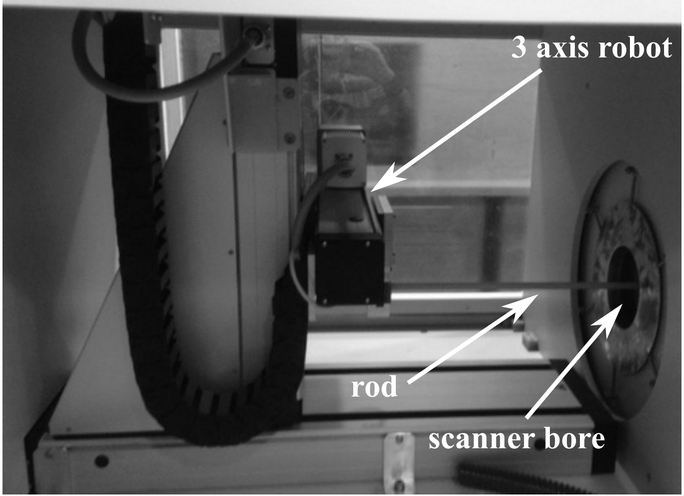
  for $n_d \in I_{N_d}$, $d = x, y, z$ and $N = N_x N_y N_z$.
- Thus, one row of the system matrix $\boldsymbol{S}$ also represents an image (in 2D) or volume (in 3D).



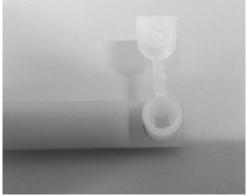
How to determine the System Matrix

- Physical modeling of $\boldsymbol{S}$ is challenging (Relaxation effects, unknown parameters).
- Therefore, system matrix $\boldsymbol{S}$ is usually explicitly measured using a robot.
- The delta sample is a voxel filled with MNP and can be mathematically represented as a unit vector $\boldsymbol{e}_j$ where $j \in I_N$.
- Since $\boldsymbol{S}\boldsymbol{e}_j = \boldsymbol{u}_j = \boldsymbol{S}_{\cdot,j}$ the calibration measurement picks the j-th column of $\boldsymbol{S}$.

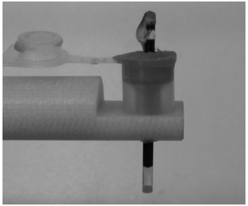
How to determine the System Matrix



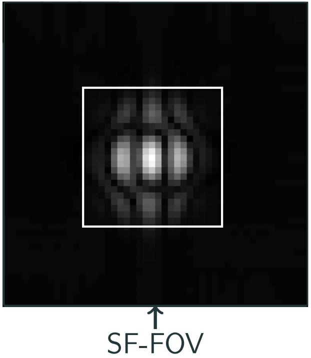
3D delta sample

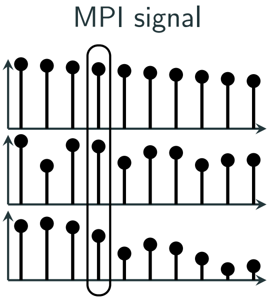


2D delta sample



MPI System Matrix

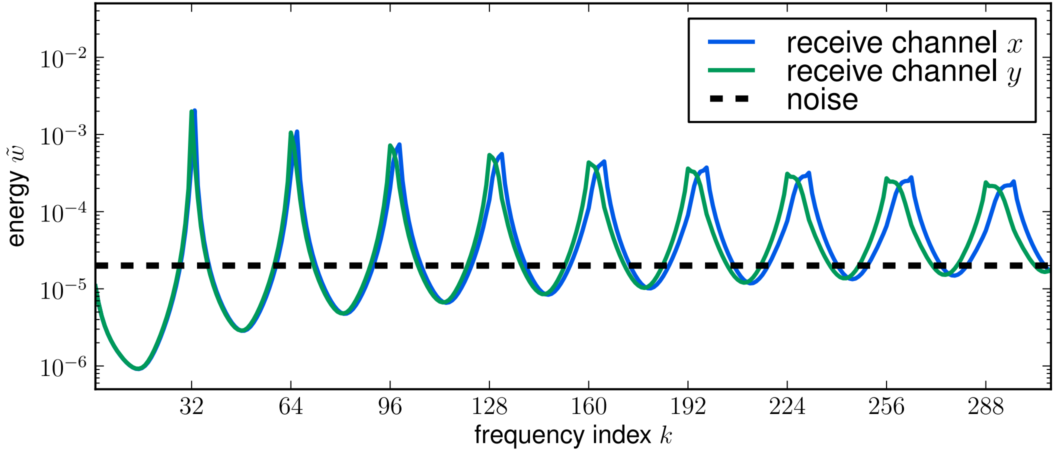




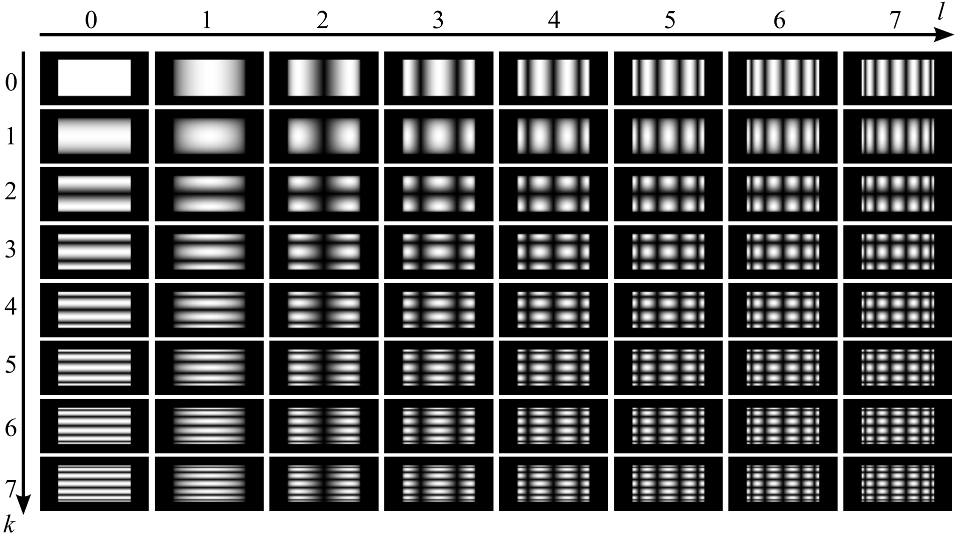
Rahmer et al. *BMC Med. Imag.* **9** (2009).

MPI System Matrix (2D)

MPI System Matrix Row Energy (2D)



Tensor products of Chebyshev polynomials



Setting - Algebraic Reconstruction

$$\hat{u}_k = \int_V s_k(\boldsymbol{r})c(\boldsymbol{r})\mathrm{d}^3 r$$

discretization

$$\boldsymbol{u} = \boldsymbol{S}\boldsymbol{c}$$

size of $\boldsymbol{S}$ $\approx$30GB for 30^3 positions

filtering

$$\boldsymbol{u}_{\text{red}} = \boldsymbol{S}_{\text{red}}\boldsymbol{c}$$

reconstruction

3-10GB

$$\operatorname{argmin}_{\boldsymbol{c}} \|\boldsymbol{S}_{\text{red}}\boldsymbol{c} - \boldsymbol{u}_{\text{red}}\|_2^2 + \lambda\|\boldsymbol{c}\|_2^2$$

reconstruction time increases with matrix size

Summary

- MPI is a tracer based imaging method exploiting the non-linear magnetization behavior of magnetic nanoparticles
- It applies different magnetic fields to achieve signal and spatial encoding
- Image reconstruction is done by solving a linear system of equations using regularization methods